Review for New 5.5, 5.6, 6,1, 6.2

Use the linearization formula to approximate the value of $f(x) = sinx$ for $x = \frac{\pi}{5}$ when $a = \frac{\pi}{6}$.

Find the real value and find the error. Explain the error.

$L(x) = f(a) + f'(a)(x - a)$ $\qquad$ $f(x) = sinx$ $\qquad$ $f'(x) = cosx$

$$L\left(\frac{\pi}{5}\right) = sin\frac{\pi}{6} + f'(\frac{\pi}{6})(\frac{\pi}{5} - \frac{\pi}{6})$$

$$L\left(\frac{\pi}{5}\right) = \frac{1}{2} + \frac{\sqrt{3}}{2}\left(\frac{\pi}{5} - \frac{\pi}{6}\right) = .591$$

$$f\left(\frac{\pi}{5}\right) = .588$$

The error is .003.

Use the linearization formula to approximate the value of $\sqrt[3]{126}$.

Find the real value and find the error. Explain the error.

$y = \sqrt[3]{x}$ $\qquad$ $y' = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3x^{\frac{2}{3}}}$ $\qquad$ $x = 126$ $\quad$ $a = 125$

$L(x) = f(a) + f'(a)(x - a)$

$$L(126) = 5 + \frac{1}{75}(1) = 5\frac{1}{75} = 5.013$$

$$f(126) = 5.013$$

The error is 0 using three place notation.

Use Newton's Method to find the zero of $y = x^3 - 2x + 3$ starting at $x = 1$ and using the method three times. What is the error?

$$f(x) = x^3 - 2x + 3 \quad f(1) = 2 \quad f'(x) = 3x^2 - 2 \qquad f'(1) = 1$$

$$y - 2 = 1(x - 1)$$

$$-2 = x - 1$$

$$-1 = x$$

$$f(x) = x^3 - 2x + 3 \quad f(-1) = 4 \quad f'(x) = 3x^2 - 2 \qquad f'(-1) = 1$$

$$y - 4 = 1(x + 1)$$

$$-4 = x + 1$$

$$x = -5$$

$$f(x) = x^3 - 2x + 3 \quad f(-5) = -112 \quad f'(x) = 3x^2 - 2 \qquad f'(-5) = 73$$

$$y + 112 = 73(x + 5)$$

$$112 = 73(x + 5)$$

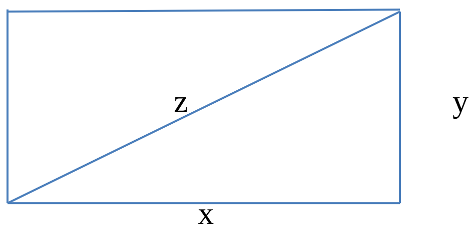
$$\frac{112}{73} = x + 5$$

$$x = -3.507$$

Actual zero is $x = -1.893$. The error is 1.634.

What is the differential of $y = \ln(x+3)$ if $x = 4\ and\ dx = .2$?

$$y' = \frac{1}{x+3} \qquad \frac{dy}{dx} = \frac{1}{x+3} \qquad dy = \frac{1}{x+3}dx = \frac{1}{7}(.2) = \frac{2}{70} = \frac{1}{35}$$



$x = 24, z = 26, x' = 2, y' = 3$

What is z'?

$$x^2 + y^2 = z^2$$

$$24^2 + y^2 = 26^2$$

$$y = 10$$

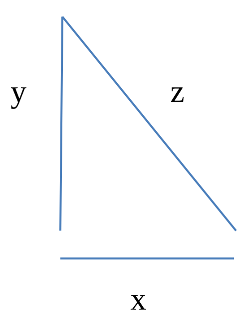
$$x^2 + y^2 = z^2$$

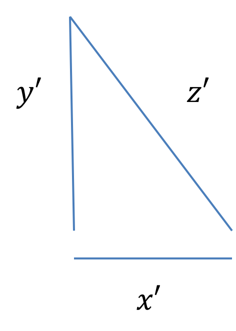
$$2xx' + 2yy' = 2zz'$$

$$2(24)2 + 2(10)(3) = 2(26)z'$$

$$96 + 60 = 52z' \qquad z' = \frac{156}{52} = 3$$

A 30 foot ladder is 24 feet from the wall and falling at a rate of 3 ft/sec. How fast is it moving away from the wall? What is the rate of the angle between the ladder and the ground?





$$x^2 + y^2 = z^2$$

$$x^2 + y^2 = z^2$$

$$24^2 + y^2 = 30^2 \qquad 2xx' + 2yy' = 2zz'$$

$$y = 18 \qquad 2(24)(x') + 2(18)(-3) = 0$$

$$48x' - 108 = 0$$

$$x' = \frac{108}{48} = 2.25$$

$$sin\theta = \frac{y}{z} \qquad sin\theta = \frac{y}{30} \qquad 30sin\theta = y \qquad 30cos\theta\theta' = y'$$

$$30\left(\frac{24}{30}\right)\theta' = -3 \qquad 24\vartheta' = -3 \qquad \vartheta' = -\frac{1}{8} \;\; rad/sec$$

A conical reservoir with vertex down is leaking water at a rate of $24\pi \frac{m^3}{hour}$. If the initial height is 12 m and the radius is 6 m, find how fast the radius and height are changing when the height is 5.

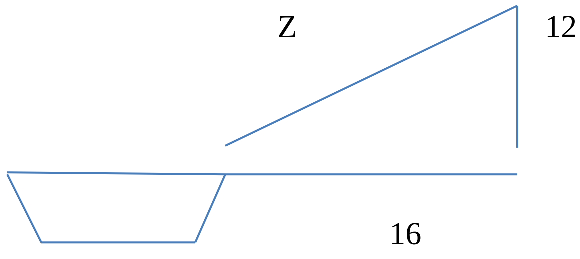
$h = 2r$ If the height is 5, the radius is 5/2

$$V = \pi r^2 h = \pi r^2(2r) = 2\pi r^3$$

$$V' = 6\pi r^2 r' \qquad -24\pi = 6\pi(\frac{5}{2})^2 r' \qquad -24\pi = \frac{6\pi r\prime(25)}{4}$$

$$-96\pi = 150\pi r' \qquad r' = -\frac{96}{150} = -\frac{16}{25}$$

A dinghy (small boat) is being pulled into shore with a rope from a 12 foot dock at a rate of 3 ft/sec. When the boat is 16 feet from the dock, how fast is the boat approaching the dock?



$$x^2 + y^2 = z^2 \qquad z = 20$$

$$2xx' + 2yy' = 2zz'$$

$$2(16)x' + 2(12)(0) = 2(20)(-3)$$

$$32x' = -120 \qquad x' = -\frac{120}{32} = -\frac{15}{4}$$

The boat is approaching the dock at $\frac{15}{4} ft/sec$.

Approximate the area between the curve and the x-axis using the LRAM, RRAM, MRAM with $f(x) = x^3 + 5x^2 + 7x + 10$ with four rectangles or trapezoids or the interval [1, 5]. Use your calculator.

$x =$	1	2	3	4	5	1.5	2.5	3.5	4.5
$y =$	23	52	103	182	295	35.125	74.375	138.625	233.875

LRAM $\quad 23(1) + 52(1) + 103(1) + 182(1) = 360$

RRAM $\quad 295(1) + 182(1) + 103(1) + 52(1) = 632$

MRAM $\quad 482$

Express the limit as a definite integral. $\quad \lim_{n\to\infty} \sum_{k=1}^{n} 2{c_k}^3 \Delta x, \quad [-2, 7]$

$$\int_{-2}^{7} (2x^3)dx$$

Use the area and the integrand to solve the integral.

$$\int_{-4}^{4} |2x - 1| dx$$

Graph $y = |2x - 1|$ You form two triangles.

$$\frac{1}{2}\left(\frac{9}{2}\right)(9) + \frac{1}{2}\left(\frac{7}{2}\right)(7) = \frac{81}{4} + \frac{49}{4} = \frac{130}{4} = \frac{65}{2}$$

$$\int_{-3}^{3} \left(7 + \sqrt{9 - x^2}\right) dx$$

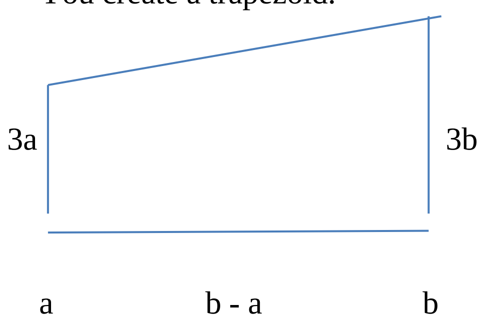
The area is a rectangle $(7)(6) = 42$.

The area of the semi-circle $\frac{\pi r^2}{2} = \pi \frac{9}{2} = \frac{9\pi}{2}$

$42 + \frac{9\pi}{2}$

$\int_a^b 3x dx$ with $0 < a < x < b$.

You create a trapezoid.



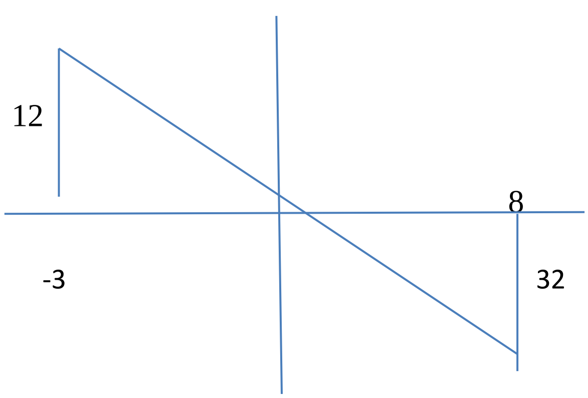
$A = \frac{1}{2}h(b_1 + b_2)$

$A = \frac{1}{2}(b - a)(3a + 3b)$

Find the total area under the curve $y = -4x$ from [-3, 8].

Total area is making all areas positive and adding them together.

You do not consider area under the x-axis as negative.



$A = \frac{1}{2}bh + \frac{1}{2}bh$

$A = \frac{1}{2}(3)(12) + \frac{1}{2}(8)(32) = 146$