

Calculus 6.2 – 5, 7.2 – 7.3 Review

Find the average value of the function $f(x) = x^2 + 6x - 2$ on the interval $[-2, 1]$. No calculator.

$$\frac{1}{1+2} \int_{-2}^1 (x^2 + 6x - 2) dx = \frac{1}{3} \left(\frac{x^3}{3} + \frac{6x^2}{2} - 2x \right) \Big|_{-2}^1 =$$

$$\frac{1}{3} \left(\frac{1}{3} + 3(4) - 2(1) \right) - \frac{1}{3} \left(-\frac{8}{3} + 12 + 4 \right) = \frac{1}{3} \left(\frac{1}{3} + 12 - 2 + \frac{8}{3} - 12 - 4 \right) = -4$$

Integrate using anti-derivatives (no calculator).

$$\int_1^6 7 dx$$

$$\int_1^e \frac{1}{x} dx$$

$$\int_1^4 -\frac{1}{x^2} dx$$

$$7x|_1^6 = 42 - 7 = 35 \quad \ln|x||_1^e = \ln e - \ln 1 = 1 \quad \frac{1}{x}|_1^4 = \frac{1}{4} - \frac{1}{1} = -\frac{3}{4}$$

What are the two parts of the Fundamental Theorem of Calculus?

1) If $y = \int_{f(x)}^{g(x)} h(t)dt$, $y' = h(g(x))g'(x) - h(f(x))f'(x)$

2) If F is the anti-derivative of f , then $\int_a^b f(x)dx = F(b) - F(a)$

Find $\frac{dy}{dx}$.

$$y = \int_0^x e^{5t} dt, \quad y' =$$

$$\int_0^x e^{5t} dt, \quad y' = e^{5x}$$

$$y = \int_{\sqrt[3]{\pi^2}}^{\ln x} t^3 dt,$$

$$y' = \frac{(\ln x)^3}{x}$$

$$y = \int_{x^5}^{e^x} \cos t dt, \quad y' =$$

$$e^x \cos e^x - 5x^4 \cos x^5$$

If $y' = 5$, $y'' = -3$, and y is the number of gallons in a tank, the gallons are

increasing at a decreasing rate.

If you integrate a rate of mosquitoes, you will get

the change in the number of mosquitos.

Evaluate the integral.

$$\int \sec^2 x e^{\tan x} dx$$

Evaluate the integral.

$$\int \sec^2 x e^{\tan x} dx \quad u = \tan x \quad \int e^u du = e^u + C \quad e^{\tan x} + C$$

$$\frac{du}{dx} = \sec^2 x$$

$$du = \sec^2 x dx$$

$$\int 4x(2x^2 + 19)^{13} dx$$

$$\int 4x(2x^2+19)^{13}dx \quad u=2x^2+19 \qquad \int u^{13}du=\frac{u^{14}}{14}+C$$

$$\frac{du}{dx}=4x \qquad \qquad \frac{(2x^2+19)^{14}}{14}+C$$

$$du=4xdx$$

$$\int \frac{2x+7}{x^2+7x-13}dx$$

$$\int \frac{2x+7}{x^2+7x-13} dx \quad u = x^2 + 7x - 13 \quad \int \frac{du}{u} = \ln|u| + C$$

$$\frac{du}{dx} = 2x + 7 \quad \ln|x^2 + 7x - 13| + C$$

$$du = (2x + 7)dx$$

$$\int (2x+5)(2x^2+10x-1)^{12} dx$$

$$\int (2x+5)(2x^2+10x-1)^{12} dx$$

$$u = 2x^2 + 10x - 1 \quad \frac{du}{dx} = 4x + 10 \quad du = (4x + 10)dx \quad du = 2(2x + 5)dx \quad \frac{du}{2} = (2x + 5)dx$$

$$\int \frac{du}{2} (u)^{12} = \frac{1}{2} \int u^{12} du = \frac{1}{2} \cdot \frac{u^{13}}{13} + C = \frac{u^{13}}{26} + C = \frac{(2x^2 + 10x - 1)^{13}}{26} + C$$

$$\int \frac{1}{x^2 + 25} dx =$$

$$\int \frac{1}{x^2 + 25} dx = \int \frac{\frac{1}{25}}{\frac{x^2}{25} + \frac{25}{25}} dx = \frac{1}{25} \int \frac{1}{(\frac{x}{5})^2 + 1} dx$$

$$u = \frac{x}{5} \quad \frac{du}{dx} = \frac{1}{5} \quad 5du = dx$$

$$\frac{1}{25} \int \frac{5du}{u^2 + 1} = \frac{1}{5} \int \frac{du}{u^2 + 1} = \frac{1}{5} \tan^{-1} u + C = \frac{1}{5} \tan^{-1} \left(\frac{x}{5} \right) + C$$

3000 Problem

$$\int \frac{1}{1 + \sqrt[3]{x+1}} dx$$

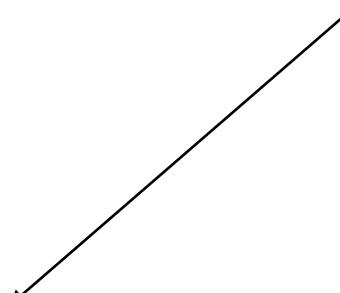
3000 Problem

$$\int \frac{1}{1+\sqrt[3]{x+1}} dx \quad u = \sqrt[3]{x+1} \quad \int \frac{3u^2 du}{1+u} \quad \text{Do long division.}$$

$$u^3 = x + 1$$

$$3u^2 \frac{du}{dx} = 1$$

$$3u^2 du = dx$$



$$u + 1 \quad \overline{)3u^2 \quad + 0u \quad + 0}$$

$$\underline{3u^2 \quad + 3u}$$

$$-3u$$

$$\underline{-3u \quad - 3}$$

$$3$$

$$\int (3u - 3 + \frac{3}{u+1}) du \quad \frac{3u^2}{2} - 3u + 3\ln|u+1| + C$$

$$\frac{\frac{3(x+1)^{\frac{2}{3}}}{2} - 3(x+1)^{\frac{1}{3}} + 3\ln|(x+1)^{\frac{1}{3}} + 1|}{2} + C$$

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Substitution Way

$$\int_1^2 x^2 \sqrt{2-x} dx$$

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The Tabular Way

$$\int_1^2 x^2 \sqrt{2-x} dx$$

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Substitution Way

$$\int_1^2 x^2 \sqrt{2-x} dx \quad u = 2 - x \quad u - 2 = -x$$

$$\frac{du}{dx} = -1 \quad -u + 2 = x$$

$$du = -dx \quad 2 - u = x$$

$$-du = dx$$

$$-\int (2-u)^2 \sqrt{u} du = -\int (4-4u+u^2) \sqrt{u} du = -\int \left(4u^{\frac{1}{2}} - 4u^{\frac{3}{2}} + u^{\frac{5}{2}}\right) du$$

$$\left(-\frac{4u^{\frac{3}{2}}}{3} + \frac{4u^{\frac{5}{2}}}{5} - \frac{u^{\frac{7}{2}}}{7}\right) = -\frac{8}{3}u^{\frac{3}{2}} + \frac{8}{5}u^{\frac{5}{2}} - \frac{2}{7}u^{\frac{7}{2}}$$

$$-\left(\frac{8}{3}(2-x)^{\frac{3}{2}} - \frac{8}{5}(2-x)^{\frac{5}{2}} + \frac{2}{7}(2-x)^{\frac{7}{2}}\right) \Big|_1^2$$

$$-\left(\frac{8(0^{\frac{3}{2}})}{3} - \frac{8(0^{\frac{5}{2}})}{5} + \frac{2(0^{\frac{7}{2}})}{7}\right) - \left(-\left(\frac{8(1^{\frac{3}{2}})}{3} + \frac{8(1^{\frac{5}{2}})}{5} - \frac{2(1^{\frac{7}{2}})}{7}\right)\right) =$$

$$-(0) - \frac{8}{3} + \frac{8}{5} - \frac{2}{7} = \frac{142}{105} = 1.3524$$

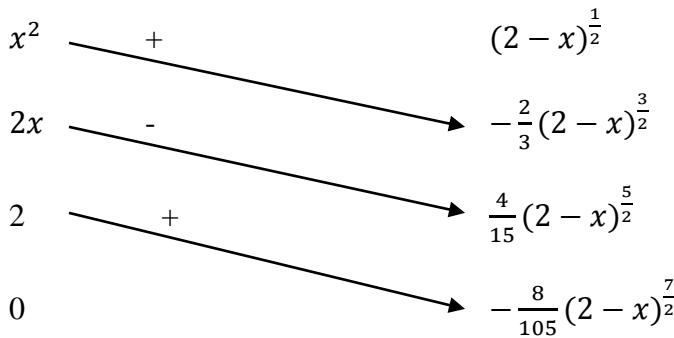
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The Tabular Way

$$\int_1^2 x^2 \sqrt{2-x} dx$$

Differentiate

Integrate



$$-\frac{2x^2}{3}(2-x)^{\frac{3}{2}} - \frac{8x}{15}(2-x)^{\frac{5}{2}} - \frac{16}{105}(2-x)^{\frac{7}{2}} \Big|_1^2 = 0 - \left(-\frac{2}{3} - \frac{8}{15} - \frac{16}{105}\right) = \frac{142}{105} = 1.3524$$

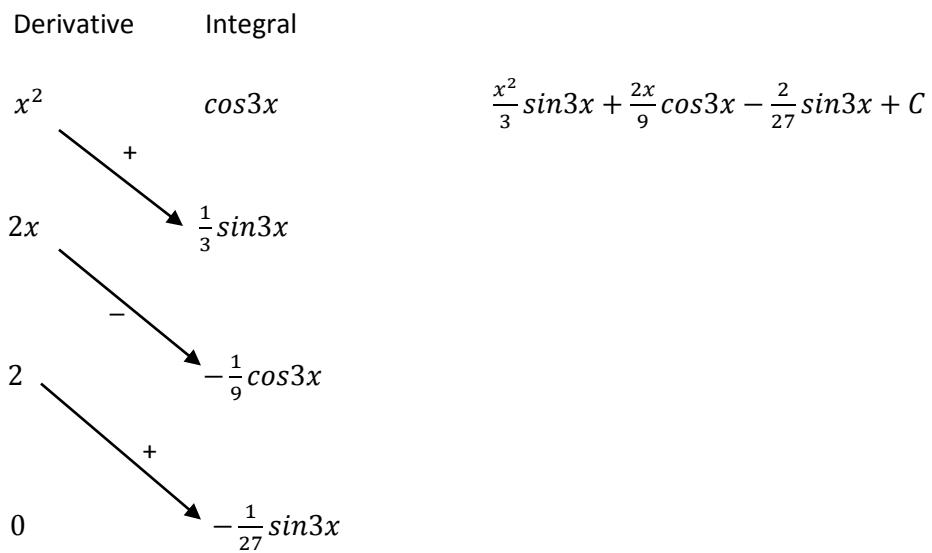
$$\int \frac{1}{1+x^2}dx \quad \int \frac{x}{1+x^2}dx \qquad \int \frac{x}{(1+x^2)^7}dx \qquad \int \frac{x+11}{1+x^2}dx$$

$$\int \frac{1}{1+x^2} dx \quad \int \frac{x}{1+x^2} dx \quad \int \frac{x}{(1+x^2)^7} dx \quad \int \frac{x+11}{1+x^2} dx$$

$$\tan^{-1}x + C \quad \frac{1}{2}\ln(1+x^2) + C \quad -\frac{1}{12}(1+x^2)^{-6} + C \quad \frac{1}{2}\ln(1+x^2) + 11\tan^{-1}x + C$$

$$\int x^2 \cos 3x dx$$

$$\int x^2 \cos 3x dx \quad \text{Tabular}$$



$$\int \ln x dx$$

$$\int \ln x dx$$

VOODOO

$$u = \ln x \qquad \qquad v' = 1$$

$$u = \frac{1}{x} \qquad \qquad v = x$$

$$uv - \int v du = x \ln x - \int x \left(\frac{1}{x}\right) dx = x \ln x - \int 1 dx = x \ln x - x + C$$

$$\int \sin^{-1} x dx$$

$$\int \sin^{-1} x dx$$

VOODOO

$$u = \sin^{-1} x \quad v' = 1$$

$$u = \frac{1}{\sqrt{1-x^2}} \quad v = x$$

$$uv - \int v du = x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx$$

$$u = 1 - x^2$$

$$\frac{du}{dx} = -2x$$

$$du = -2dx$$

$$\frac{du}{-2} = dx$$

$$\int -\frac{du}{2} \frac{1}{\sqrt{u}} = -\frac{1}{2} \int u^{-\frac{1}{2}} du = -\frac{\frac{1}{2}u^{\frac{1}{2}}}{\frac{1}{2}} + C = -u^{\frac{1}{2}} + C = -(1-x^2)^{\frac{1}{2}} + C$$

$$x \sin^{-1} x + (1-x^2)^{\frac{1}{2}} + C$$

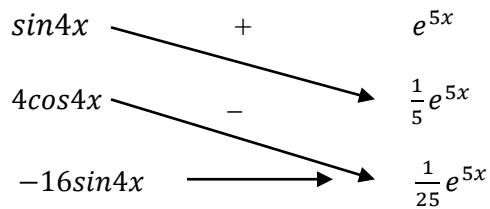
$$\int \sin 4x e^{5x} dx$$

$$\int \sin 4x e^{5x} dx$$

GREID

Derivative

Integral



$$\int \sin 4x e^{5x} dx = \frac{e^{5x} \sin 4x}{5} - \frac{e^{5x} \cos 4x}{25} - \int \frac{16e^{5x} \sin 4x}{25}$$

$$\int \sin 4x e^{5x} dx + \int \frac{16e^{5x} \sin 4x}{25} = \left(\frac{e^{5x} \sin 4x}{5} - \frac{e^{5x} \cos 4x}{25} \right) + C$$

$$\frac{41}{25} \int \sin 4x e^{5x} dx = \left(\frac{e^{5x} \sin 4x}{5} - \frac{e^{5x} \cos 4x}{25} \right) + C$$

$$\int \sin 4x e^{5x} dx = \frac{25e^{5x}}{41} \left(\frac{\sin 4x}{5} - \frac{\cos 4x}{25} \right) + C$$

Extra Fun

$$\int \sec x dx$$

$$\int \sec x dx \cdot \frac{\sec x + \tan x}{\sec x + \tan x} = \int \frac{\sec^2 x + \sec x \tan x}{\sec x + \tan x} dx$$

$$u = \sec x + \tan x \quad \frac{du}{dx} = \sec x \tan x + \sec^2 x$$

$$du = (\sec x \tan x + \sec^2 x) dx$$

$$\int \frac{du}{u} = \ln|u| + C = \ln|\sec x + \tan x| + C$$

More Extra Fun

$$\int \frac{1}{x^2 + 4x + 5} dx$$

$$\int \frac{1}{x^2 + 4x + 5} dx = \int \frac{1}{x^2 + 4x + 4 + 1} dx = \int \frac{1}{(x+2)^2 + 1} dx$$

$$u = x + 2 \quad \frac{du}{dx} = 1 \quad du = dx$$

$$\int \frac{1}{u^2 + 1} du = \tan^{-1} u + C = \tan^{-1}(x+2) + C$$

Over the Top Fun

$$\int \frac{4x}{4x^4 + 12x^2 + 10} dx$$

$$\int \frac{4x}{4x^4 + 12x^2 + 10} dx$$

$$\int \frac{4x}{(2x^2 + 3)^2 + 1} dx \quad u = 2x^2 + 3 \quad \frac{du}{dx} = 4x \quad du = 4dx$$

$$\int \frac{du}{u^2 + 1} = \tan^{-1} u + C = \tan^{-1}(2x^2 + 3) + C$$