Calculus   6.2 – 5, 7.2 – 7.3 Review

Find the average value of the function $f(x) = x^2 + 6x - 2$ on the interval [-2, 1]. No calculator.

$$\frac{1}{1+2}\int_{-2}^{1}(x^2+6x-2)dx = \frac{1}{3}(\frac{x^3}{3}+\frac{6x^2}{2}-2x)\Big|_{-2}^{1} =$$

$$\frac{1}{3}\left(\frac{1}{3}+3(4)-2(1)\right)-\frac{1}{3}\left(-\frac{8}{3}+12+4\right)=\frac{1}{3}\left(\frac{1}{3}+12-2+\frac{8}{3}-12-4\right)=-4$$

Integrate using anti-derivatives (no calculator).

$\int_1^6 7dx$ $\int_1^e \frac{1}{x}dx$ $\int_1^4 -\frac{1}{x^2}dx$

$7x\Big|_1^6 = 42 - 7 = 35$ $\qquad ln|x|\Big|_1^e = lne - ln1 = 1$ $\qquad \frac{1}{x}\Big|_1^4 = \frac{1}{4} - \frac{1}{1} = -\frac{3}{4}$

What are the two parts of the Fundamental Theorem of Calculus?

1) If $y = \int_{f(x)}^{g(x)} h(t)dt, y' = h(g(x))g'(x) - h(f(x))f'(x)$

2) If F is the anti-derivative of , then $\int_a^b f(x)dx = F(b) - F(a)$

Find $\frac{dy}{dx}$.

$$y = \int_0^x e^{5t}dt, \qquad y' =$$

$$\int_0^x e^{5t}dt, \qquad y' = e^{5x}$$

$$y = \int_{\sqrt[3]{\pi^2}}^{lnx} t^3 dt,$$

$$y' = \frac{(lnx)^3}{x}$$

$$y = \int_{x^5}^{e^x} costdt, \quad y' =$$

$$e^x cose^x - 5x^4 cosx^5$$

If $y' = 5, y'' = -3$, and y is the number of gallons in a tank, the gallons are

increasing at a decreasing rate.

If you integrate a rate of mosquitoes, you will get the change in the number of mosquitos.

Evaluate the integral.

$\int sec^2 x e^{tan x} dx$

Evaluate the integral.

$\int sec^2 x e^{tanx} dx$ $\quad$ $u = tanx$ $\quad$ $\int e^u du = e^u + C$ $\quad$ $e^{tanx} + C$

$\frac{du}{dx} = sec^2 x$

$du = sec^2 x dx$

$$\int 4x(2x^2 + 19)^{13} dx$$

$\int 4x(2x^2+19)^{13}dx \qquad u = 2x^2+19 \qquad \int u^{13}du = \frac{u^{14}}{14}+C$

$\frac{du}{dx} = 4x \qquad \frac{(2x^2+19)^{14}}{14}+C$

$du = 4xdx$

$\int \frac{2x+7}{x^2+7x-13}dx$

$$\int \frac{2x+7}{x^2+7x-13}dx \qquad u = x^2 + 7x - 13 \qquad \int \frac{du}{u} = ln|u| + C$$

$$\frac{du}{dx} = 2x + 7 \qquad ln|x^2 + 7x - 13| + C$$

$$du = (2x + 7)dx$$

$$\int (2x + 5)(2x^2 + 10x - 1)^{12}dx$$

$$\int (2x+5)(2x^2+10x-1)^{12}dx$$

$u = 2x^2 + 10x - 1$ $\quad \frac{du}{dx} = 4x + 10$ $\quad du = (4x+10)dx$ $\quad du = 2(2x+5)dx$ $\quad \frac{du}{2} = (2x+5)dx$

$$\int \frac{du}{2}(u)^{12} = \frac{1}{2}\int u^{12}du = \frac{1}{2}\cdot\frac{u^{13}}{13} + C = \frac{u^{13}}{26} + C = \frac{(2x^2+10x-1)^{13}}{26} + C$$

$$\int \frac{1}{x^2+25}dx =$$

$$\int \frac{1}{x^2+25}dx = \int \frac{\frac{1}{25}}{\frac{x^2}{25}+\frac{25}{25}}dx = \frac{1}{25}\int \frac{1}{(\frac{x}{5})^2+1}dx$$

$$u = \frac{x}{5} \quad \frac{du}{dx} = \frac{1}{5} \quad 5du = dx$$

$$\frac{1}{25}\int \frac{5du}{u^2+1} = \frac{1}{5}\int \frac{du}{u^2+1} = \frac{1}{5}tan^{-1}u + C \ = \ \frac{1}{5}tan^{-1}(\frac{x}{5}) + C$$

3000 Problem

$$\int \frac{1}{1+\sqrt[3]{x+1}}dx$$

3000 Problem

$\int \frac{1}{1+\sqrt[3]{x+1}}dx$ $\quad u = \sqrt[3]{x+1}$ $\quad \int \frac{3u^2du}{1+u}$ Do long division.

$u^3 = x+1$

$3u^2\frac{du}{dx} = 1$

$3u^2du = dx$

$$\begin{array}{r|llll} & 3u & -3 & & \\ \hline u+1 & 3u^2 & +0u & +0 & \\ & \underline{3u^2} & \underline{+3u} & & \\ & & -3u & & \\ & & \underline{-3u} & \underline{-3} & \\ & & & 3 & \end{array}$$

$3u - 3 + \frac{3}{u+1}$

$\int (3u - 3 + \frac{3}{u+1})du$ $\quad \frac{3u^2}{2} - 3u + 3ln|u+1| + C$

$\frac{3(x+1)^{\frac{2}{3}}}{2} - 3(x+1)^{\frac{1}{3}} + 3ln\left|(x+1)^{\frac{1}{3}} + 1\right| + C$

Virginia Tech Substitution Way

$\int_1^2 x^2\sqrt{2-x}dx$

Virginia Tech The Tabular Way

$\int_1^2 x^2\sqrt{2-x}dx$

Virginia Tech Substitution Way

$\int_1^2 x^2\sqrt{2-x}dx$ $u = 2 - x$ $u - 2 = -x$

$\frac{du}{dx} = -1$ $-u + 2 = x$

$du = -dx$ $2 - u = x$

$-du = dx$

$$-\int (2-u)^2\sqrt{u}du = -\int (4-4u+u^2)\sqrt{u}\, du = -\int \left(4u^{\frac{1}{2}} - 4u^{\frac{3}{2}} + u^{\frac{5}{2}}\right) du$$

$$\left(-\frac{4u^{\frac{3}{2}}}{\frac{3}{2}} + \frac{4u^{\frac{5}{2}}}{\frac{5}{2}} - \frac{u^{\frac{7}{2}}}{\frac{7}{2}}\right) = -\frac{8}{3}u^{\frac{3}{2}} + \frac{8}{5}u^{\frac{5}{2}} - \frac{2}{7}u^{\frac{5}{2}}$$

$$-(\frac{8}{3}(2-x)^{\frac{3}{2}} - \frac{8}{5}(2-x)^{\frac{5}{2}} + \frac{2}{7}(2-x)^{\frac{5}{2}})\Big|_1^2$$

$$-\left(\frac{8(0^{\frac{3}{2}})}{3} - \frac{8(0^{\frac{3}{2}})}{5} + \frac{2(0^{\frac{3}{2}})}{7}\right) - (-\left(-\frac{8(1^{\frac{3}{2}})}{3} + \frac{8(1^{\frac{3}{2}})}{5} - \frac{2(1^{\frac{3}{2}})}{7}\right)) =$$

$$-(0) - \frac{8}{3} + \frac{8}{5} - \frac{2}{7} = \frac{142}{105} = 1.3524$$

Virginia Tech The Tabular Way

$\int_1^2 x^2\sqrt{2-x}dx$

Differentiate		Integrate
x^2	+	$(2-x)^{\frac{1}{2}}$
$2x$	-	$-\frac{2}{3}(2-x)^{\frac{3}{2}}$
2	+	$\frac{4}{15}(2-x)^{\frac{5}{2}}$
0		$-\frac{8}{105}(2-x)^{\frac{7}{2}}$

$$-\frac{2x^2}{3}(2-x)^{\frac{3}{2}} - \frac{8x}{15}(2-x)^{\frac{5}{2}} - \frac{16}{105}(2-x)^{\frac{7}{2}}\Big|_1^2 = 0 - \left(-\frac{2}{3} - \frac{8}{15} - \frac{16}{105}\right) = \frac{142}{105} = 1.3524$$

$$\int \frac{1}{1+x^2}\,dx \quad \int \frac{x}{1+x^2}\,dx \qquad \int \frac{x}{(1+x^2)^7}\,dx \qquad \int \frac{x+11}{1+x^2}\,dx$$

$\int \frac{1}{1+x^2}dx$ $\int \frac{x}{1+x^2}dx$ $\int \frac{x}{(1+x^2)^7}dx$ $\int \frac{x+11}{1+x^2}dx$

$tan^{-1}x + C$ $\frac{1}{2}\ln(1+x^2) + C$ $-\frac{1}{12}(1+x^2)^{-6} + C$ $\frac{1}{2}\ln(1+x^2) + 11tan^{-1}x + C$

$\int x^2 cos3xdx$

$\int x^2 cos3xdx$ Tabular

Derivative		Integral
x^2		$cos3x$
	$+$	
$2x$		$\frac{1}{3}sin3x$
	$-$	
2		$-\frac{1}{9}cos3x$
	$+$	
0		$-\frac{1}{27}sin3x$

$$\frac{x^2}{3}sin3x + \frac{2x}{9}cos3x - \frac{2}{27}sin3x + C$$

$\int lnxdx$

$\int lnxdx$ VOODOO

$u = lnx \qquad v' = 1$

$u = \frac{1}{x} \qquad v = x$

$$uv - \int vdu = xlnx - \int x\left(\frac{1}{x}\right)dx = xlnx - \int 1dx = xlnx - x + C$$

$\int sin^{-1}xdx$

$\int sin^{-1}xdx$ VOODOO

$u = sin^{-1}x \qquad v' = 1$

$u = \frac{1}{\sqrt{1-x^2}} \qquad v = x$

$$uv - \int vdu = xsin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}dx$$

$u = 1 - x^2$

$\frac{du}{dx} = -2x$

$du = -2dx$

$\frac{du}{-2} = dx$

$$\int -\frac{du}{2}\frac{1}{\sqrt{u}} = -\frac{1}{2}\int u^{-\frac{1}{2}}du = -\frac{\frac{1}{2}u^{\frac{1}{2}}}{\frac{1}{2}} + C = -u^{\frac{1}{2}} + C = -(1-x^2)^{\frac{1}{2}} + C$$

$xsin^{-1}x + (1-x^2)^{\frac{1}{2}} + C$

$\int sin4xe^{5x}dx$

$\int sin4xe^{5x}dx$ GREID

Derivative		Integral
$sin4x$	$+$	e^{5x}
$4cos4x$	$-$	$\frac{1}{5}e^{5x}$
$-16sin4x$		$\frac{1}{25}e^{5x}$

$$\int sin4xe^{5x}dx = \frac{e^{5x}sin4x}{5} - \frac{e^{5x}cos4x}{25} - \int \frac{16e^{5x}sin4x}{25}$$

$$\int sin4xe^{5x}dx + \int \frac{16e^{5x}sin4x}{25} = \left(\frac{e^{5x}sin4x}{5} - \frac{e^{5x}cos4x}{25}\right) + C$$

$$\frac{41}{25}\int sin4xe^{5x}dx = \left(\frac{e^{5x}sin4x}{5} - \frac{e^{5x}cos4x}{25}\right) + C$$

$$\int sin4xe^{5x}dx = \frac{25e^{5x}}{41}\left(\frac{sin4x}{5} - \frac{cos4x}{25}\right) + C$$

Extra Fun

$$\int secxdx$$

$$\int secxdx \cdot \frac{secx + tanx}{secx + tanx} = \int \frac{sec^2x + secxtanx}{secx + tanx} dx$$

$$u = secx + tanx \qquad \frac{du}{dx} = secxtanx + sec^2x$$

$$du = (secxtanx + sec^2x)dx$$

$$\int \frac{du}{u} = ln|u| + C = ln|secx + tanx| + C$$

More Extra Fun

$$\int \frac{1}{x^2 + 4x + 5} dx$$

$$\int \frac{1}{x^2 + 4x + 5} dx = \int \frac{1}{x^2 + 4x + 4 + 1} dx = \int \frac{1}{(x+2)^2 + 1} dx$$

$$u = x + 2 \qquad \frac{du}{dx} = 1 \qquad du = dx$$

$$\int \frac{1}{u^2 + 1} du = tan^{-1}u + C = tan^{-1}(x + 2) + C$$

Over the Top Fun

$$\int \frac{4x}{4x^4 + 12x^2 + 10} dx$$

$$\int \frac{4x}{4x^4 + 12x^2 + 10} dx$$

$$\int \frac{4x}{(2x^2 + 3)^2 + 1} dx \qquad u = \ 2x^2 + 3 \qquad \frac{du}{dx} = 4x \qquad du = 4dx$$

$$\int \frac{du}{u^2 + 1} = tan^{-1}u + C = ta\,n^{-1}(2x^2 + 3) + C$$