Polar One

Points

Name three other polar points for the given point.

1) $(3, \pi)$ $(-3, 0)$ $(3, -\pi)$ $(3, 3\pi)$

2) $(-2, \frac{\pi}{3})$ $\left(2, \frac{4\pi}{3}\right)$ $\left(-2, -\frac{5\pi}{3}\right)$ $(2, -\frac{2\pi}{3})$

Rectangular points are in the form $(x, y)$. Polar points are in the form $(r, \theta)$.

$$x = rcos\theta, y = rsin\theta, r = \sqrt{x^2 + y^2} \quad tan\theta = \frac{y}{x}$$

3) Convert $\left(2, \frac{5\pi}{6}\right)$ to rectangular coordinates.

$$x = 2\cos\left(\frac{5\pi}{6}\right) = 2\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}, \; y = 2\sin\left(\frac{5\pi}{6}\right) = 2\left(\frac{1}{2}\right) = 1$$

$$(-\sqrt{3}, 1)$$

4) Convert$(3, -3)$ to polar coordinates.

This point is in the 4th quadrant on the 45 degree angle.

$$r = \sqrt{(3)^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}$$

$(3\sqrt{2}, \frac{\pi}{4})$

5) Convert the following equations to polar form.

a) $y = 3$

$rsin\theta = 3$

$r = 3/sin\theta$

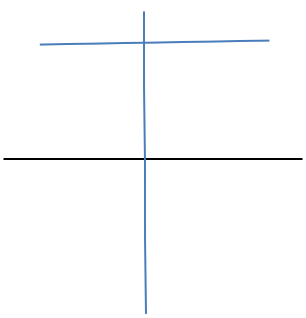
$r = 3csc\theta$

b) $x^2 + y^2 = 25$

$r = 5$

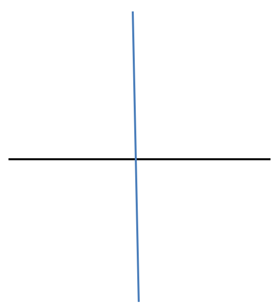
6) Convert the following equations to rectangular form, and sketch the graph. No calculator.

a) $rsin\theta = 5$



$y = 5$

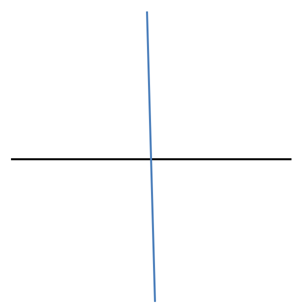
b) $r = 2cos\theta$



$r^2 = 2rcos\theta$

$x^2 + y^2 = 2x$

c) $\theta = \frac{2\pi}{3}$

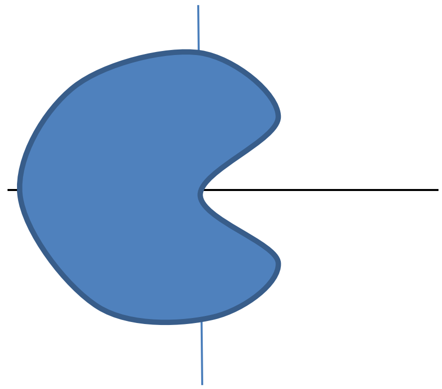


$tan\theta = \tan(\frac{2\pi}{3})$

$\frac{y}{x} = \sqrt{3}$

$y = \sqrt{3}x$

7) Use your calculator, in polar mode, to graph $r = 1 - cos\theta$.



8) If $r = 1 - cos\theta$, use information about $x$ & $y$ and substitute for $r$ in terms of $\theta$, and write the polar curve in parametric form.

$$r = 1 - cos\theta$$

$$x = rcos\theta \qquad y = rsin\theta$$

$$\frac{x}{cos\theta} = r \qquad \frac{y}{sin\theta} = r$$

$$\frac{x}{cos\theta} = 1 - cos\theta \qquad \frac{y}{sin\theta} = 1 - cos\theta$$

$$x = cos\theta - \cos^2\theta \qquad y = sin\theta - sin\theta cos\theta$$

9) Use your calculator and graph the equation in parametric form and see if it looks the same as # 7.

10) Find $\frac{dy}{dx}$, which is the slope of the polar curve, parametrically.

$$x = cos\theta - \cos^2\theta \qquad y = sin\theta - sin\theta cos\theta$$

$$\frac{dx}{d\theta} = -sin\theta + 2cos\theta sin\theta \qquad \frac{dy}{d\theta} = cos\theta - \cos^2\theta + \sin^2\theta$$

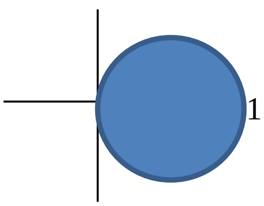
$$\frac{dy}{dx} = \frac{cos\theta - \cos^2\theta + \sin^2\theta}{-sin\theta + 2cos\theta sin\theta}$$

11) Find the slope of the curve $r = 1 - cos\theta$ at $\theta = \frac{\pi}{2}$.

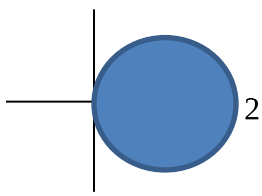
$$\frac{dy}{dx} = \frac{cos\theta - \cos^2\theta + \sin^2\theta}{-sin\theta + 2cos\theta sin\theta} = \frac{cos\frac{\pi}{2} - \cos^2\frac{\pi}{2} + \sin^2\frac{\pi}{2}}{-sin\frac{\pi}{2} + 2cos\frac{\pi}{2}sin\frac{\pi}{2}} = \frac{1}{-1} = 1$$

Graph the following using your calculator (Polar, Radian & Path style)

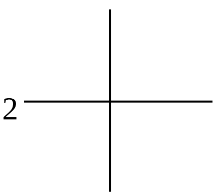
1) $r = cos\theta$



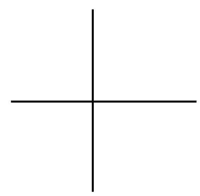

2) $r = 2cos\theta$




3) $r = 3cos\theta$



4) $r = -3cos\theta$

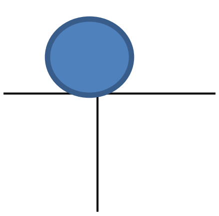


What window is needed to trace each curve out once? $(0, \pi)$

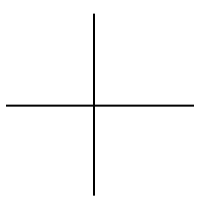
What is the basic shape of each graph? Circle

What type of symmetry is in each graph? Symmetric to the x-axis.

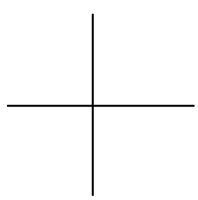
5) $r = sin\theta$



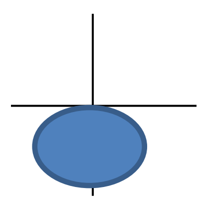
6) $r = 2sin\theta$



7) $r = 3sin\theta$



8) $r = -3sin\theta$



What window is needed to trace each curve out once? $(0, \pi)$

What is the basic shape of each graph? Circle

What type of symmetry is in each graph? To the y-axis

If the equation is $r = a\cos\theta$, the graph is a circle with diameter $a$

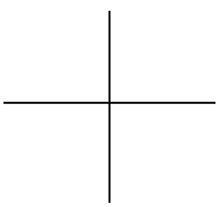
on the positive x - axis if $a > 0$ and on the negative x- axis if $a < 0$..

If the equation is $r = a\sin\theta$, the graph is a circle with diameter $a$

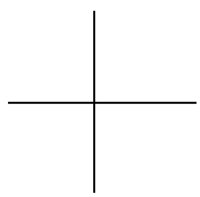
on the positive y - axis if $a > 0$ and on the negative y- axis if $a < 0$..

Graph the following using your calculator (Polar, Radian & Path style)

9) $r = 2 + 2\cos\theta$

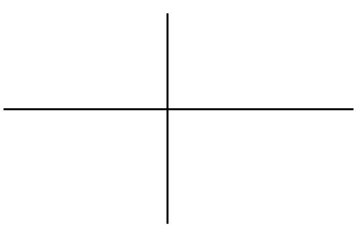


10) $r = 1 + 2\cos\theta$

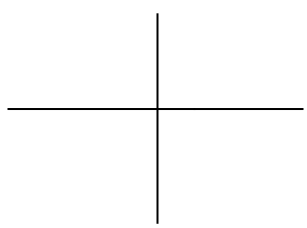


Graph the same equations in the Cartesian mode. This is called the auxiliary Cartesian graph.

9C) $y = 2 + 2\cos x$



10C) $y = 1 + 2\cos x$



What window is needed to trace each curve out once?

What shape is each curve? 9) 10)

Which graphs go though the pole?

What type of symmetry do you see?

Look at the auxiliary Cartesian # 10 graph. Change the window to $\left[0, 2\pi, \frac{\pi}{3}\right], [-2, 4, 1]$.

Go to the table and see where the graph is negative.

Look at the polar # 10 graph.

Change the window to $\theta = \left[0, 2\pi, \frac{\pi}{12}\right], x = \left[-\frac{\pi}{12}, 2\pi, \frac{\pi}{12}\right], y = [-2, 4, 1]$

Go to the table and see where the graph is negative.

What happens to the polar graph when the Cartesian graph shows that the function is negative?

It creates an inside loop.

11) $r = 2 + cos\theta$

12) $r = 2 - 4cos\theta$

11C) $y = 2 + cosx$

12C) $y = 2 - 4cosx$

What window is needed to trace out each curve once?

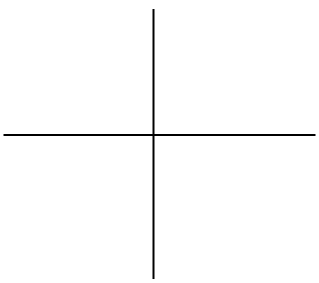
What is the shape of each curve? 11) 12)

Which graphs go through the pole?

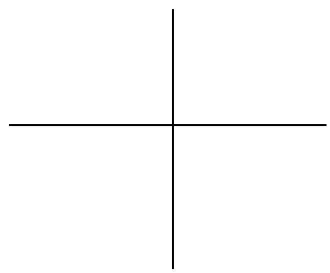
Which graphs do not go through the pole?

What happens to the polar graph when the Cartesian graph shows that the function is negative?

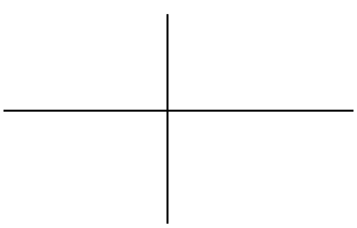
13) $r = 2 + 2sin\theta$



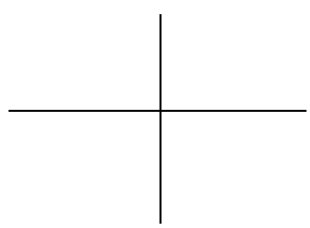
14) $r = 1 + 2sin\theta$



13C) $y = 2 + 2sinx$



14C) $y = 1 + \ 2sinx$



What window is needed to trace out each curve once?

What is the shape of each curve? 11) 12)

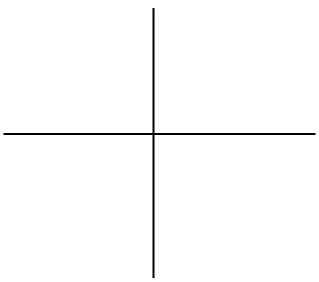
What type of symmetry do you see?

Which graphs go through the pole?

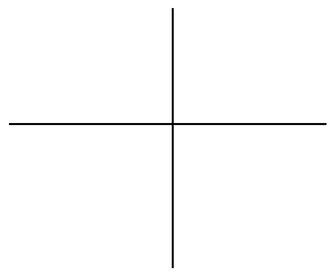
Which graphs do not go through the pole?

What happens to the polar graph when the Cartesian graph shows that the function is negative?

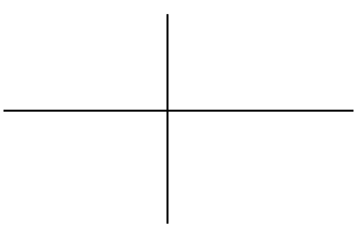
15) $r = 2 + sin\theta$



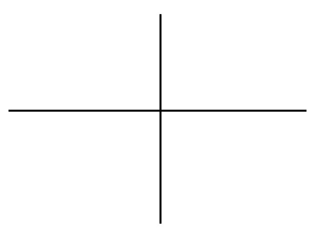
16) $r = 2 - 4sin\theta$



15C) $y = 2 + sinx$



16C) $y = 2 - 4sinx$



What window is needed to trace out each curve once?

What is the shape of each curve? 15) 16)

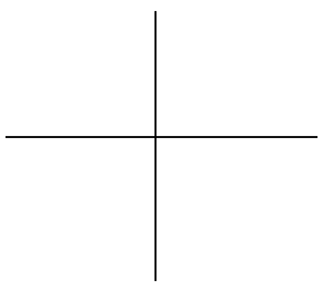
What type of symmetry do you see?

Which graphs go through the pole?

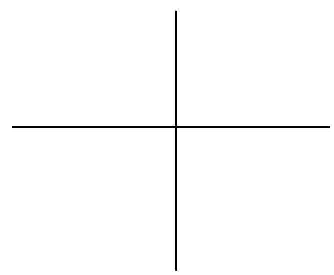
Which graphs do not go through the pole?

What happens to the polar graph when the Cartesian graph shows that the function is negative?

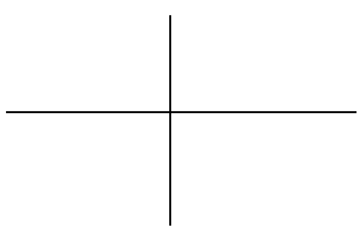
17) $r = 2cos3\theta$



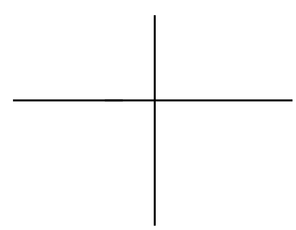
18) $r = 3cos5\theta$



19) $r = 4cos7\theta$



20) $y = -2cos3\theta$



What window is needed to trace out each curve once?

What is the shape of each curve?

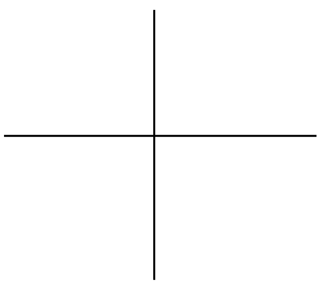
What type of symmetry do you see?

If the coefficient of $\theta$ is odd, how many petals are there?

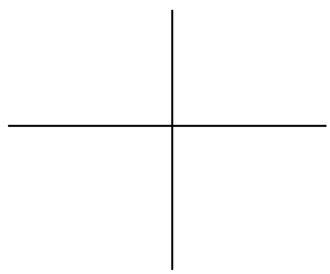
How long are the petals?

What do all of these graphs have in common?

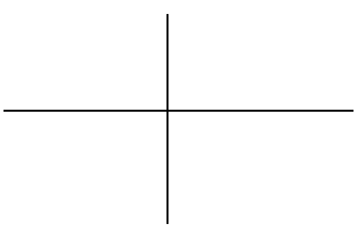
21) $r = 2sin3\theta$



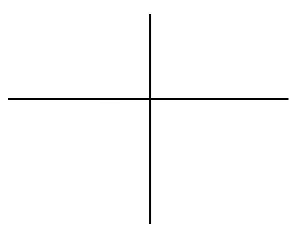
22) $r = 3sin5\theta$



23) $r = 4sin7\theta$



24) $y = -2sin3\theta$



What window is needed to trace out each curve once?

What is the shape of each curve?

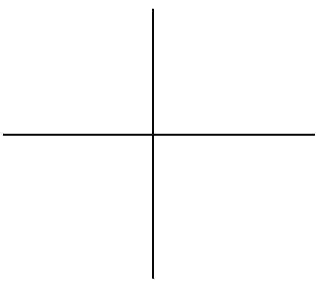
What type of symmetry do you see?

If the coefficient of $\theta$ is odd, how many petals are there?

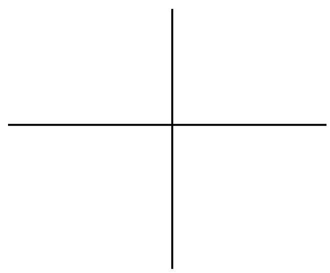
How long are the petals?

What do all of these graphs have in common?

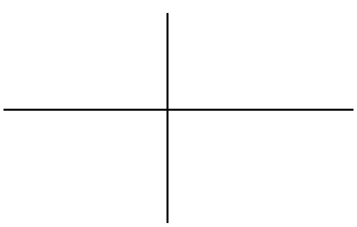
25) $r = 3cos2\theta$



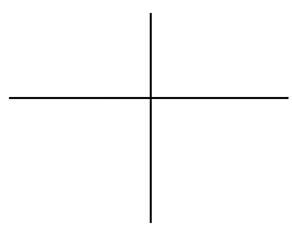
26) $r = 2cos4\theta$



27) $r = 4cos6\theta$



28) $y = -3cos2\theta$



What window is needed to trace out each curve once?

What is the shape of each curve?

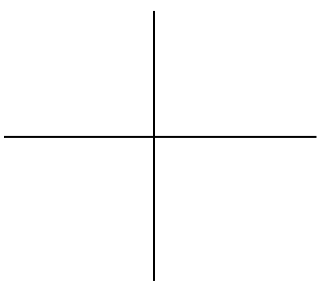
What type of symmetry do you see?

If the coefficient of $\theta$ is even, how many petals are there?

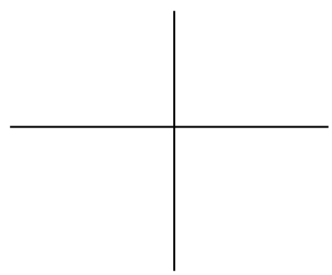
How long are the petals?

What do all of these graphs have in common?

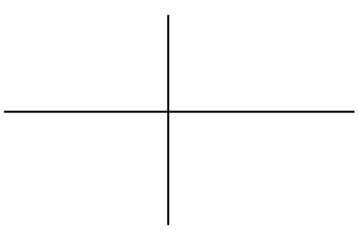
29) $r = 3sin2\theta$



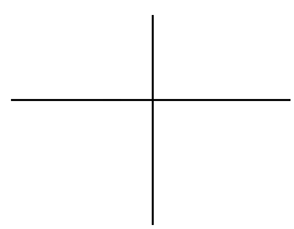
30) $r = 2sin4\theta$



31) $r = 4sin6\theta$



32) $y = -3sin2\theta$



What window is needed to trace out each curve once?

What is the shape of each curve?

What type of symmetry do you see?

If the coefficient of $\theta$ is even, how many petals are there?

How long are the petals?

What do all of these graphs have in common?

If the graph is in the form $r = \text{a}\cos(n\theta)$, there are n petals if $n$ is odd, 2n petals if $n$ is even and each petal is of length $|a|$, and there is one petal on the x axis.

If the graph is in the form $r = \text{a}\sin(n\theta)$, there are n petals if $n$ is odd, 2n petals if $n$ is even and each petal is of length $|a|$, and there is one petal on the y axis if n is odd

To find the slope of a tangent line to a polar curve $r = f(\theta)$, use the facts that

$$x = rcos\theta$$

and

$$y = rsin\theta$$

along with the product rule and chain rule, when appropriate to get

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$$

33) Find $\frac{dy}{dx}$ and the slope of the graph of the polar curve $r = 2 + 2sin\theta$ when $\theta = \pi/2$. No calculator.

$$r = 2 + 2sin\theta$$

$$x = rcos\theta = (2 + 2sin\theta)(cos\theta) = 2cos\theta + 2sin\theta cos\theta$$

$$\frac{dx}{d\theta} = -2sin\theta + 2((sin\theta)(-sin\theta) + cos\theta cos\theta)$$

$$-2sin\theta - 2\sin^2\theta + \cos^2\theta$$

$$y = rsin\theta = (2 + 2sin\vartheta)(sin\vartheta) = 2sin\vartheta + 2\sin^2\vartheta$$

$$\frac{dy}{d\theta} = 2cos\theta + 4sin\theta cos\theta$$

$$\frac{dy}{dx} = \frac{2cos\theta + 4sin\theta cos\theta}{-2sin\theta - 2\sin^2\theta + \cos^2\theta} \ at\ \theta = \frac{\pi}{4} \quad \frac{2cos\frac{\pi}{2} + 4sin\frac{\pi}{2}cos\frac{\pi}{2}}{-2sin\frac{\pi}{2} - 2\sin^2\frac{\pi}{2} + \cos^2\frac{\pi}{2}}$$

$$\frac{0+0}{-2-2} = 0$$

Graph the equation and see if the slope is 0 at $\theta = \frac{\pi}{2}$.

34) Find the points of horizontal and vertical tangency to the graph of

$r = 2 - 2cos\theta$ over $[0, 2\pi]$. No calculator.

$$r = 2 - 2cos\theta$$

$$x = rcos\theta = (2 - 2cos\theta)(cos\theta) = 2cos\theta - 2\cos^2\theta$$

$$\frac{dx}{d\theta} = -2sin\theta - 4cos\theta(-sin\theta) = -2sin\theta + 4sin\theta cos\theta$$

$$y = rsin\theta = (2 - 2cos\theta)sin\theta = 2sin\theta - 2sin\theta cos\theta$$

$$\frac{dy}{d\theta} = 2cos\theta - 2sin\theta(-sin\theta) - 2cos\theta cos\theta$$

$$\frac{dy}{d\theta} = 2cos\theta + 2\sin^2\theta - 2\cos^2\theta$$

There is a horizontal tangent when $\frac{dy}{d\theta} = 0 \,\&\, \frac{dx}{dt} \neq 0.$

$$\frac{dy}{d\theta} = -2sin\theta + 4sin\theta cos\theta = 0 \quad 2sin\theta(-1 + 2cos\theta) = 0$$

$$2sin\theta = 0 \; at \theta = 0 \,\&\, \pi \qquad -1 + 2cos\theta = 0 \quad cos\theta = \frac{1}{2} \quad at \frac{2\pi}{3} \; or \frac{4\pi}{3}$$

When $\theta = 0, x = 2cos0 - 2\cos^2 0 = 0$ $\quad y = 2sin0 - 2sin0cos0 = 0$

When $x = 0, \frac{dx}{dt} = 0$ so we do not want this point.

When $\theta = \frac{2\pi}{3}$ $, r = 2 - 2\cos\left(\frac{2\pi}{3}\right) = 2 - 2\left(-\frac{1}{2}\right) = 2 + 1 = 3.$

The point is $\left(3, \frac{2\pi}{3}\right)$.