New 9.1 Sequences

Defined explicitly means that the sequence is defined in terms of n. You can find any term with only the n value.

$$a_n = \frac{n}{5n-1}$$

Give the first 5 terms of this sequence:

n	1	2	3	4	5
$\frac{n}{5n-1}$	$\frac{1}{4}$	$\frac{2}{9}$	$\frac{3}{14}$	$\frac{4}{19}$	$\frac{5}{24}$

Defined recursively: the sequence is defined in terms of previous terms. You can only find the next term.

$$a_n = 3a_{n-1}, a_1 = 2$$

Give the first 5 terms of this sequence:

n	1	2	3	4	5
$\frac{n}{5n-1}$	2	6	18	54	162

An arithmetic sequence is where the numbers go up or down by adding or subtracting a number.

Example:

1, 4, 7, 10, 13, ……

If a is your first term and you are increasing by a value of d, write a general formula for the sequence recursively.

$$a_n = a_{n-1} + 3, a_1 = 1$$

It can also be written as explicitly as:

$a_n = a_1 + (n-1)d$ where d is the common difference

A geometric sequence is where the numbers are increasing or decreasing by a factor of the number right before it.

$1, 2, 4, 8, 16, \ldots \ldots$

This can be written recursively as $a_n = ra_{n-1}$where r is the factor being multiplied.

This can also be written explicitly as $a_n = a_1 r^{n-1}$.

Does the sequence approach any specific value?

Let's look at some arithmetic sequences:

$d = 3$ $1, 4, 7, 10, 13, \ldots ..$

$d = -6$ $1, -5, -11, -17, -23 \ldots \ldots$

Arithmetic sequences do not approach a specific value.

Let's look at some geometric sequences:

$r = 2$ $1, 2, 4, 8, 16 \ldots \ldots$

$r = -1$ $1, -1, 1, -1, 1 \ldots \ldots$

$r = \frac{1}{2}$ $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \ldots \ldots ..$

$r = -\frac{1}{3}$ $9, -3, 1, -\frac{1}{3}, \frac{1}{9} \ldots ....$

If $-1 < r < 1$, the sequence approaches a certain value.

If a sequence approaches a certain value, it converges to that value.

If a sequence does not approach a certain value, it diverges.

The best way to see if the sequence converges or diverges is to find $\lim_{n\to\infty} a_n$. This is Bill Gates.

Determine the convergence or divergence of the sequence with the given nth term. If the sequence converges, find the limit.

$a_n = \frac{2n}{n+3}$ $\lim_{n\to\infty} a_n = \frac{2n}{n} = 2$

$a_n = (-1)^n \frac{n+1}{n^2+2}$ $\lim_{n\to\infty} (-1)^n \frac{1}{n} = 0$

$a_n = (1.1)^n$ This diverges as $r = 1.1$ which is outside of $(-1, 1)$.

$a_n = \cos(\frac{n\pi}{2})$ This diverges because you write it out, it flip flops from -1 to 0 to 1 to 0.

The Sandwich Theorem for Sequences

If $\lim_{n\to\infty} a_n = \lim_{n\to\infty} c_n = L$

$a_n \le b_n \le c_n$ for all n > N, then $\lim_{n\to\infty} b_n = L$.

Show that the sequence $\{\frac{\cos n}{n}\}$ converges.

$\left|\frac{\cos n}{n}\right| \le \frac{|\cos n|}{|n|} \le \frac{1}{n}$ $\frac{-1}{n} \le \frac{\cos n}{n} \le \frac{1}{n}$

$\lim_{n\to\infty} \frac{\cos n}{n} = 0 \textit{ because } \lim_{n\to\infty} -\frac{1}{n} = \lim_{n\to\infty} \frac{1}{n} = 0$

If the limit exists, the sequence converges.

If $\lim_{n\to\infty} |a_n| = 0$, then $\lim_{n\to\infty} a_n = 0$.