New Calculus 8.2

$\int_a^b f(x)dx$ is the area between the curve and the x-axis over the interval a to b.

$\int_a^b (f(x) - g(x))dx$ is the area between the curves over the interval a to b.

Find the area between the curves $y = sec^2x$ and $y = sinx$ from $x = 0\ to\ x = \frac{\pi}{4}$.

$$\int_0^{\frac{\pi}{4}} (sec^2x - sinx)dx = tanx + cosx\Big|_0^{\frac{\pi}{4}} = 1 + \frac{\sqrt{2}}{2} - 0 - 1 = \frac{\sqrt{2}}{2}$$

Find the area between the curves $f(x) = x^2\ and\ y = 4x$ in the first quadrant.

The curves intersect at $x = 0\ and\ x = 4$.

$$\int_0^4 (4x - x^2)dx = 2x^2 - \frac{x^3}{3}\Big|_0^4 = 32 - \frac{64}{3} = \frac{32}{3}$$

Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

$2 - x^2 = -x \quad 0 = x^2 - x - 2 \quad 0 = (x - 2)(x + 1)$

The curves intersect at $x = -1, 2$.

$$\int_{-1}^2 (2 - x^2 + x)dx = 2x - \frac{x^3}{3} + \frac{x^2}{2}\Big|_{-1}^2 \quad 4 - \frac{8}{3} + 2 - \left(-2 + \frac{1}{3} + \frac{1}{2}\right) = 8 - 3 - \frac{1}{2} = 4\frac{1}{2}$$

Find the area in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$.

Draw the graph.

Method # 1

$$\int_0^2 \sqrt{x}dx + \int_2^4 (\sqrt{x} - (x-2))dx = \frac{2}{3}x^{\frac{3}{2}}\Big|_0^2 + (\frac{2}{3}x^{\frac{3}{2}} - \frac{x^2}{2} + 2x)\Big|_2^4 = \frac{10}{3}$$

Method # 2

$$\int_0^4 \sqrt{x}dx - \int_2^4 (x-2)dx = \frac{2}{3}x^{\frac{3}{2}}\Big|_0^4 - \frac{x^2}{2} + 2x)\Big|_2^4 = \frac{10}{3}$$

Method # 3

$$\int_0^4 \sqrt{x}dx - triangle = \frac{2}{3}x^{\frac{3}{2}}\Big|_0^4 - \frac{1}{2}(2)(2) = \frac{10}{3}$$

Method # 4