New Calculus 8.1

If you integrate a velocity curve, you get the displacement (change in distance).

If you integrate the absolute value of the velocity, you get the total distance traveled.

If you integrate an acceleration curve, you get the change in velocity.

If you integrate a rate, you get a change in amount.

$\int_a^b v(t)dt =$ the net change in distance over the time a to b

$\int_a^b |v(t)|dt =$ the total distance traveled over the time a to b

$s(a) + \int_a^b v(t)dt =$ the position at time b

a) Determine when the particle is moving to the right, left or stopped.
b) Find the particle's displacement for the given time interval.

If $s(0) = 3$, what is the particle's final position?

c) Find the total distance traveled by the particle.

$v(t) = 6sin3t, \quad 0 \le t \le \frac{\pi}{2}$

a) $v(t) = 6sin3t = 0 \qquad at\ t = 0, \frac{\pi}{3}$

Stopped at $t = 0, \frac{\pi}{3}$

$0 < t < \frac{\pi}{3} \quad v(t) > 0$ so the particle is moving to the right.

$\frac{\pi}{3} < t \le \frac{\pi}{2} \quad v(t) < 0$ so the particle is moving to the left

b) $\int_0^{\frac{\pi}{2}} 6sin3tdt = -2cos3t|_0^{\frac{\pi}{2}} \quad -2cos\frac{3\pi}{2} + 2cos0 = 2$

$2 + 3 = 5$

c) $\int_0^{\frac{\pi}{3}} 6sin3tdt - \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} 6sin3tdt = -2cos3t|_0^{\frac{\pi}{3}} + 2cos3t|_{\frac{\pi}{3}}^{\frac{\pi}{2}} =$

$$= -2cos\pi + 2cos0 + 2cos\frac{3\pi}{2} - 2cos\pi$$
$$= 2 + 2 + 0 + 2 = 6$$

$v(t) = \dfrac{t}{1+t^2}, \quad 0 \le t \le 3$

$a)\ v(t) = \frac{t}{1+t^2} = 0 \quad at\ t = 0$

Stopped at $t = 0$.

$0 < t \le 3 \quad v(t) > 0$ so the particle is moving to the right

The particle is never moving to the left.

b) $\int_0^3 \frac{t}{1+t^2}\,dt = \frac{1}{2}ln10 - \frac{1}{2}ln1 = \frac{1}{2}ln10 = 1.15$

$1.15 + 3 = 4.5$

c) $\int_0^3 \frac{t}{1+t^2}\,dt = \frac{1}{2}ln10 - \frac{1}{2}ln1 = \frac{1}{2}ln10 = 1.15$