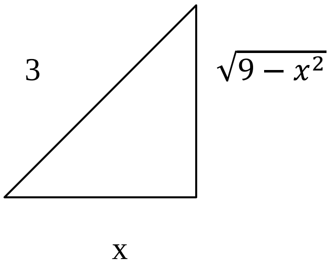
7.5.5 Trig Substitutions

$\int \frac{dx}{x^2\sqrt{9-x^2}}$



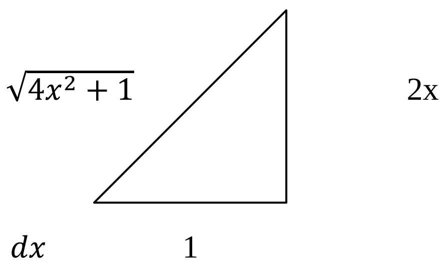
$cos\theta = \frac{x}{3}$ $\quad 3cos\theta = x$ $\quad -3sin\theta d\theta = dx$

$sin\theta = \frac{\sqrt{9-x^2}}{3}$ $\quad 3sin\theta = \sqrt{9-x^2}$

$\int \frac{-3sin\theta d\theta}{(3cos\theta)^2(3sin\theta)} d\theta = \int \frac{-3}{9cos^2\theta} d\theta$ $\quad -\frac{1}{3}\int sec^2\theta d\theta = -\frac{1}{3}tan\theta + C$

$-\frac{1}{3}tan\theta + C = -\frac{1}{3}\frac{\sqrt{9-x^2}}{x} + C$

$\int \frac{1}{\sqrt{4x^2+1}} dx$



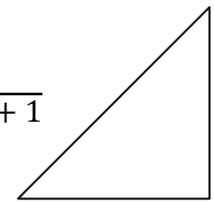
$tan\theta = \frac{2x}{1}$ $\quad \frac{1}{2}tan\theta = x$ $\quad \frac{1}{2}sec^2\theta = \frac{dx}{d\theta}$ $\quad \frac{1}{2}sec^2\theta d\theta = dx$

$sec\theta = \sqrt{4x^2+1}$

$\int \frac{1}{\sqrt{4x^2+1}} dx = \int \frac{\frac{1}{2}sec^2\theta d\theta}{sec\vartheta} = \frac{1}{2}\int sec\theta d\theta = \frac{1}{2}ln|sec\theta + tan\theta| + C$

$\frac{1}{2}ln\left|\sqrt{4x^2+1} + 2x\right| + C$

$\int \frac{dx}{(x^2+1)^{\frac{3}{2}}}$



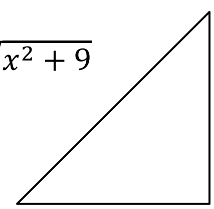
$sec\theta = \sqrt{x^2+1} \quad sec^3\theta = (\sqrt{x^2+1})^3$

$tan\theta = x \qquad sec^2\theta = \frac{dx}{d\theta} \qquad sec^2\theta d\theta = dx$

$$\int \frac{sec^2\theta d\theta}{sec^3\theta} = \int \frac{1}{sec\theta} d\theta = \int cos\theta d\theta = sin\theta + C$$

$sin\theta + C = \frac{x}{\sqrt{x^2+1}} + C$

$\int \frac{x}{\sqrt{x^2+9}} dx$



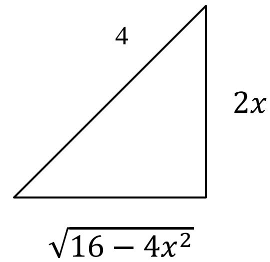
$tan\theta = \frac{x}{3} \qquad 3tan\theta = x \qquad 3sec^2\theta = \frac{dx}{d\theta} \qquad 3sec^2\theta d\theta = dx$

$sec\theta = \frac{\sqrt{x^2+9}}{3} \qquad 3sec\theta = \sqrt{x^2+9}$

$$\int \frac{x}{\sqrt{x^2+9}} dx = \int \frac{3tan\theta(3sec^2\theta)d\theta}{3sec\theta} = \int 3sec\theta tan\theta d\theta = 3sec\theta + C$$

$3sec\theta + C = \sqrt{x^2+9} + C$

$\int x\sqrt{16-4x^2}\,dx$



$$sin\theta = \frac{2x}{4} \qquad 4sin\theta = 2x \qquad 2sin\theta = x \qquad 2cos\theta = \frac{dx}{d\theta} \qquad 2cos\theta = dx$$

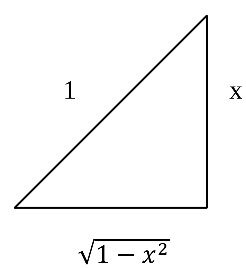
$$cos\theta = \frac{\sqrt{16-4x^2}}{4} \qquad 4cos\theta = \sqrt{16-4x^2}$$

$$\int x\sqrt{16-4x^2}\,dx = \int (2sin\theta)(4cos\theta)(2cos\theta)d\theta \qquad = 16\int sin\theta cos^2\theta d\theta$$

Let $u = cos\theta$ $\quad \frac{du}{d\theta} = -sin\theta \quad du = -sin\theta d\theta\ \theta \quad -du = sin\theta d\theta \qquad -16\int u^2 du$

$$\frac{u^3}{3} + C = \frac{cos^3\theta}{3} + C = \frac{1}{3}\left(\frac{\sqrt{16-4x^2}}{4}\right)^3 + C$$

$\int \frac{\sqrt{1-x^2}}{x^4}dx$



$sin\theta = x \quad cos\theta = \frac{dx}{d\theta} \qquad cos\theta d\theta = dx$

$$\int \frac{cos\theta}{sin^4\theta}\cdot cos\theta d\theta \qquad \int \frac{cos^2\theta}{sin^4\theta}d\theta = \int \frac{cot^2\theta}{sin^2\theta}d\theta = \int cot^2\theta csc^2\theta d\theta$$

Let $u = cot\theta \quad \frac{du}{d\theta} = -csc^2\theta \qquad du = -csc^2\theta d\theta \quad -du = csc^2\theta d\theta$

$$-\int u^2 du = -\frac{u^3}{3} + C \quad -\frac{(cot\theta)^3}{3} + C$$

$$-\frac{1}{3}\left(\frac{\sqrt{1-x^2}}{x}\right)^3 + C$$