7.4 Differential Equations

$$\frac{dy}{dx} = ky$$

What would this mean? The change in y is a number k times the y value.

If you have a larger y value, your increase would be more.

What real life situation does this model?

The rich get richer.

Solve for y.

$$\frac{dy}{dx} = ky$$

$$\frac{dy}{y} = kdx \qquad \int \frac{dy}{y} = \int kdx \qquad ln|y| + C_1 = kx + C_2$$

$$ln|y| = kx + C \quad |y| = e^{kx+C} \qquad y = e^{kx}e^{C} \qquad y = Ce^{kx} \qquad y = y_0e^{kx}$$

Does this formula look familiar?

This is the half life formula that was used in Math Analysis. It was also used for interest compounded on a daily basis.

This is called a differential equation. You are given a derivative and you want to solve for the original equation.

$$\frac{dy}{dx} = \frac{2x}{y} \qquad ydy = 2xdx \qquad \int ydy = \int 2xdx$$

$$\frac{y^2}{2} = x^2 + c \quad y^2 = 2x^2 + C \qquad y = \pm\sqrt{2x^2 + C}$$

$$\frac{dy}{dx} = xy \qquad \frac{dy}{y} = xdx \qquad \int \frac{dy}{y} = \int xdx$$

$$ln|y| = \frac{x^2}{2} + C \qquad y = e^{\frac{x^2}{2}+C} \qquad y = Ce^{\frac{x^2}{2}}$$

$$y^2dy + x^3dx = 0 \quad y^2dy = -x^3dx$$

$$\int y^2dy = -\int x^3dx \quad \frac{y^3}{3} = -\frac{x^4}{4} + C \qquad y^3 = -\frac{3x^4}{4} + C \quad y = \sqrt[3]{-\frac{3x^4}{4} + C}$$

$$\frac{dy}{dx} = x^8y^{12} \qquad dy = x^8y^{12}dx \qquad \frac{dy}{y^{12}} = x^8dx$$

$$\int \frac{dy}{y^{12}} = \int x^8dx \qquad \frac{y^{-11}}{-11} = \frac{x^9}{9} + C \quad y^{-11} = \frac{-11x^9}{9} + C \qquad y = \frac{1}{\sqrt[11]{\frac{-11x^9+9C}{9}}} = \sqrt[11]{\frac{9}{-11x^9+9C}}$$

$$\frac{2dy}{dx} = \frac{y(x+1)}{x} \qquad \frac{dy}{y} = \frac{(x+1)dx}{2x}$$

$$\int \frac{dy}{y} = \int \frac{(x+1)dx}{2x} \qquad ln|y| = \frac{1}{2}\int \frac{(x+1)dx}{x} \qquad ln|y| = \frac{1}{2}\int \frac{x}{x}dx + \frac{1}{2}\int \frac{1}{x}dx$$

$$ln|y| = \frac{1}{2}\int dx + \frac{1}{2}\int \frac{1}{x}dx = \frac{1}{2}x + \frac{1}{2}ln|x| + C$$

$$y = e^{\frac{1}{2}x+\frac{1}{2}ln|x|+C} = e^{\frac{1}{2}x}e^{ln\sqrt{|x|}}e^{C} = C|x|e^{\frac{1}{2}x}$$

$$\frac{dy}{dx}lnx - \frac{y}{x} = 0 \quad \frac{dy}{dx}xlnx - y = 0 \quad \frac{dy}{dx}xlnx = y \quad dy(xlnx) = ydx$$

$$\frac{dy}{y} = \frac{dx}{xlnx} \quad \int \frac{dy}{y} = \int \frac{dx}{xlnx}$$

$ln|y| = \int \frac{dx}{xlnx}$ Use a u substitution and $u = lnx$ $\frac{du}{dx} = \frac{1}{x}$ $du = \frac{x}{dx}$

$$ln|y| = \int \frac{du}{u} \quad ln|y| = ln|u| + C \quad ln|y| = ln|lnx| + C$$

$$y = e^{ln|lnx|+C} = |lnx|e^C = C|lnx|$$

$$\frac{dy}{dx} + 2y = 6 \quad x = 0 \, when \, y = 1 \quad \frac{dy}{dx} = 6 - 2y \quad \frac{dy}{6-2y} = dx$$

$$\int \frac{dy}{6-2y} = \int dx \quad u = 6 - 2y \quad \frac{du}{dy} = -2 \quad \frac{du}{-2} = dy$$

$$\int \frac{du}{-2} \cdot \frac{1}{u} = \int dx \quad -\frac{1}{2}ln|u| = x + C \quad ln|u| = -2x - 2C$$

$$ln|6 - 2y| = -2x - 2C \quad ln|6 - 2| = -2(0) - 2C \quad ln4 = -2C \quad C = -\frac{ln4}{2}$$

$$ln|6 - 2y| = -2x - 2\left(-\frac{ln4}{2}\right) \quad ln|6 - 2y| = -2x + ln4$$

$$ln|6 - 2y| - ln4 = -2x \quad ln\frac{6-2y}{4} = -2x \quad \frac{6-2y}{4} = e^{-2x}$$

$$6 - 2y = 4e^{-2x} \quad -2y = -6 + 4e^{-2x} \quad y = 3 - 2e^{-2x}$$