7.3 Integration by Parts Three Ways

First Way - Basic VOODOO

$(uv)' = uv' + vu'$

$\int (uv)'' = \int uv' + \int vu'$

$uv = \int uv' + \int vu'$

$uv - \int vu' = \int uv'$ ULTRA VIOLET VOODOO DOLLS

The purpose of integration by parts is to not integrate the problem as you see it but to change the form and then integrate it.

$\int xcosxdx$

Find one function that you can integrate and one function that you can take a derivative of.

$u = x \qquad v' = cosx$

$u' = 1 \qquad v = sinx$

$uv - \int vu' = \int uv'$

$xsinx - \int sinx(1)dx = xsinx + cosx + C$

$\int lnxdx$

Find one function that you can integrate and one function that you can take a derivative of.

$u = lnx \qquad v' = 1$

$u' = \frac{1}{x} \qquad v = x$

$uv - \int vu' = \int uv'$

$xlnx - \int x\frac{1}{x}dx = xlnx - \int dx = xlnx - x + C$

$\int sin^{-1}xdx$

Find one function that you can integrate and one function that you can take a derivative of.

$u = sin^{-1}x \qquad v' = 1$

$u' = \frac{1}{\sqrt{1-x^2}} \qquad v = x$

$uv - \int vu' = \int uv'$

$xsin^{-1}x - \int \frac{x}{\sqrt{1-x^2}}dx =$

Use a u-substitution where $u = 1 - x^2 \quad \frac{du}{dx} = -2x \quad du = -2xdx \quad \frac{du}{-2} = xdx$

$xsin^{-1}x - \int \frac{du}{-2\sqrt{u}} = xsin^{-1}x + \frac{1}{2}\int u^{-\frac{1}{2}}du = xsin^{-1}x + \frac{1}{2}\frac{u^{\frac{1}{2}}}{\frac{1}{2}} = xsin^{-1}x + u^{\frac{1}{2}} + C$

$xsin^{-1}x + (1-x^2)^{\frac{1}{2}} + C$

Second Way - Tabular

THIS IS THE WRONG WAY TO DO THIS PROBLEM!!!

$\int x^2e^xdx$

Find one function that you can integrate and one function that you can take a derivative of.

$u = x^2 \qquad v^{'} = e^x$

$u^{'} = 2x \qquad v = e^x$

$uv - \int vu' = \int uv^{'}$

$x^2e^x - 2\int xe^xdx$

Find one function that you can integrate and one function that you can take a derivative of.

$u = x \qquad v^{'} = e^x$

$u^{'} = 1 \qquad v = e^x$

$uv - \int vu' = \int uv^{'}$

$x^2e^x - 2(xe^x - \int e^xdx) = x^2e^x - 2xe^x + 2 + C$

THIS IS THE RIGHT WAY TO DO THIS PROBLEM!!!

To use Tabular, you need to have one function that will differentiate to 0 and the other function can be integrated.

Derivative		Integral
x^2	+	e^x
$2x$	-	e^x
2	+	e^x
0		e^x

$$x^2e^x - 2xe^x + 2e^x + C$$

Third Way GREID

If the problems has e^x & sinx $or\ cosx$, you can do a variation of Tabular.

$$\int sinxe^x dx$$

Derivative		Integral
$sinx$	+	e^x
$cosx$	-	e^x
$-sinx$	→	e^x

$$\int sinxe^x dx = sinxe^x - cosxe^x - \int sinxe^x dx$$

$$2\int sinxe^x = sinxe^x - cosxe^x$$

$$\int sinxe^x = \frac{1}{2}(sinxe^x - cosxe^x) + C$$

Try it again.

$$\int e^{3x} cos4xdx$$

You have to have the sin or cos in the derivative column.

Take the derivative and integral twice, do the diagonals twice (alternating signs), and then integrate straight across (using the signs that are given). The first integral and the last integral should have the same variables but with different coefficients. Add or subtract the right integral to the left integral, combining the fractions. Divide the right side of the equation by the reciprocal of the fraction in front of the integral.

Derivative		Integral
$cos4x$	+	e^{3x}
$-4sin4x$	-	$\frac{1}{3}e^{3x}$
$-16cos4x$		$\frac{1}{9}e^{3x}$

$$\int e^{3x} cos4xdx = \frac{1}{3}e^{3x}\cos\ (4x) + \frac{4}{9}e^{3x}\sin(4x) - \frac{16}{9}\int e^{3x} cos4x$$

$$\frac{9}{9}\int e^{3x} cos4xdx = \frac{1}{3}e^{3x}\cos(4x) + \frac{4}{9}e^{3x}\sin(4x) - \frac{16}{9}\int e^{3x} cos4x$$

$$\frac{25}{9}\int e^{3x} cos4xdx = \frac{1}{3}e^{3x}\cos(4x) + \frac{4}{9}e^{3x}\sin(4x)$$

$$\int e^{3x} cos4xdx = \frac{9}{25}(\frac{1}{3}e^{3x}\cos(4x) + \frac{4}{9}e^{3x}\sin(4x)) + C$$

$$\int e^{3x} cos4xdx = e^{3x}(\frac{3}{25}\cos(4x) + \frac{4}{25}\sin(4x)) + C$$

When method do you use?

Look for tabular all of the time. If one term can be differentiated until it goes to 0 and the other term can be integrated.

If you see sin or cos and e^x, use GREID.

Use straight up VOODOO as a last resort. If the problem has a lnx in it, you will using this method.