New 7.2 U-Substitution

$\int 2x\left(e^{x^2}\right)dx \qquad u = x^2 \quad \frac{du}{dx} = 2x \quad du = 2xdx$

$$\int \frac{du}{dx} e^u = e^u + C = \ e^{x^2} + C$$

$\int sec^2 x e^{tanx} dx \qquad u = tanx \quad \frac{du}{dx} = sec^2 x \quad du = sec^2 xdx$

$$\int e^u du = e^u + C = \ e^{tanx} + C$$

$\int \frac{e^{sin^{-1}x}}{\sqrt{1-x^2}} dx \qquad u = sin^{-1}x \quad \frac{du}{dx} = \frac{1}{\sqrt{1-x^2}} \quad du = \frac{dx}{\sqrt{1-x^2}}$

$$\int e^u du = e^u + C = \ e^{sin^{-1}x} + C$$

$\int 4x(2x^2+19)^{13} dx \quad u = 2x^2 + 19 \quad \frac{du}{dx} = 4x \quad du = 4xdx$

$$\int u^{13} du = \frac{u^{14}}{14} + C \ = \ \frac{(2x^2+19)^{14}}{14} + C$$

$\int cosxsin^5 x\, dx \qquad u = sinx \quad \frac{du}{dx} = cosx \quad du = cosxdx$

$$\int u^5 du = \frac{u^6}{6} + C = \frac{sin^6 x}{6} + C$$

$\int \frac{2x+7}{x^2+7x-13} dx \qquad u = x^2 + 7x - 13 \quad \frac{du}{dx} = 2x + 7 \quad du = (2x+7)dx$

$$\int \frac{du}{u} = ln|u| + C = ln|x^2 + 7x - 13| + C$$

$$\int \frac{cosx}{sinx} dx \qquad u = sinx \quad \frac{du}{dx} = cosx \quad du = cosxdx$$

$$\int \frac{du}{u} = ln|u| + C = ln|sinx| + C$$

$$\int \frac{2x+4}{x^2+4x-6} dx \qquad u = x^2 + 4x - 6 \quad \frac{du}{dx} = 2x + 4 \quad du = (2x+4)dx$$

$$\int \frac{du}{u} = ln|u| + C = ln|x^2 + 4x - 6| + C$$

Fun Cases

$$\int \frac{1}{x^2+16} dx = \int \frac{\frac{1}{16}}{\frac{x^2}{16} + \frac{16}{16}} dx = \frac{1}{16} \int \frac{dx}{\frac{x^2}{16} + 1} = \frac{1}{16} \int \frac{dx}{(\frac{x}{4})^2 + 1}$$

$$u = \frac{x}{4} \quad \frac{du}{dx} = \frac{1}{4} \quad 4du = dx \qquad \frac{1}{16} \int \frac{4du}{u^2+1} = \frac{4}{16} \int \frac{du}{u^2+1} = \frac{1}{4} \int \frac{du}{u^2+1} =$$

$$\frac{1}{4} tan^{-1}u + C = \frac{1}{4} tan^{-1}\frac{x}{4} + C$$

3000 Problem

$$\int \frac{1}{1+\sqrt[4]{x+1}}dx \qquad u = \sqrt[4]{x+1} \qquad u^4 = x+1 \quad u^4 - 1 = x$$

$$4u^3 \frac{du}{dx} = 1 \qquad 4u^3 du = dx$$

$\int \frac{4u^3 du}{1+u}$ Long Division

$$4u^2 - 4u + 4$$

$$u+1 \;)\; 4u^3 + 0u^2 + 0u + 0$$

$$\underline{4u^3 + 4u^2}$$

$$-4u^2 + 0u$$

$$\underline{-4u^2 - 4u}$$

$$4u$$

$$\underline{4u + 4}$$

$$-4$$

$$\int \frac{4u^3 du}{1+u} = \int (4u^2 - 4u + 4 - \frac{4}{u+1})du$$

Integrate separately

$$\int 4u^2 du - \int 4u du + \int 4\,du - \int \frac{4}{u+1}du$$

$$4\left(\frac{u^3}{3} - \frac{u^2}{2} + u - ln|u+1|\right) + C$$

$$4\left(\frac{(x+1)^{\frac{3}{4}}}{3} - \frac{(x+1)^{\frac{1}{2}}}{2} + (x+1)^{\frac{1}{4}} - ln\left|(x+1)^{\frac{1}{4}} + 1\right|\right) + C$$

This is a great problem.

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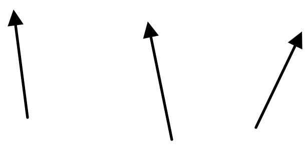
$$\int_{-1}^{0} x^2\sqrt{1+x}dx \qquad u = 1+x \quad u-1 = x \quad \frac{du}{dx} = 1 \qquad du = dx$$

$\int \ (u-1)^2 u^{\frac{1}{2}} du$ Foil it out and then multiply by $u^{\frac{1}{2}}$.

$$\int (u^2 - 2u + 1)u^{\frac{1}{2}}du \qquad \int (u^{\frac{5}{2}} - 2u^{\frac{3}{2}} + u^{\frac{1}{2}})du$$

$$\frac{u^{\frac{7}{2}}}{\frac{7}{2}} - \frac{2u^{\frac{5}{2}}}{\frac{5}{2}} + \frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2u^{\frac{7}{2}}}{7} - \frac{4u^{\frac{5}{2}}}{5} + \frac{2u^{\frac{3}{2}}}{3} = \left. \frac{2(1+x)^{\frac{7}{2}}}{7} - \frac{4(1+x)^{\frac{5}{2}}}{5} + \frac{2(1+x)^{\frac{3}{2}}}{3} \right|_{-1}^{0}$$

$$\frac{2(1)^{\frac{7}{2}}}{7} - \frac{4(1)^{\frac{5}{2}}}{5} + \frac{2(1)^{\frac{3}{2}}}{3} - 0 = \frac{2}{7} - \frac{4}{5} + \frac{2}{3} = \frac{16}{105}$$



This is a great problem. All great problems are on the test.

$$\int \frac{dx}{4-3x} \qquad u = 4-3x \qquad \frac{du}{dx} = -3 \qquad du = -3dx \qquad \frac{du}{-3} = dx$$

$$\int \frac{du}{-3u} = -\frac{1}{3}\int \frac{du}{u} = -\frac{1}{3}ln|u| + C \qquad -\frac{1}{3}ln|4-3x| + C$$

$$\int_{1}^{e} \frac{\sqrt{lnx}}{x}dx \qquad u = lnx \qquad \frac{du}{dx} = \frac{1}{x} \qquad xdu = dx \qquad \int \frac{\sqrt{u}}{x}xdu = \int \sqrt{u}du$$

$$\frac{u^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2u^{\frac{3}{2}}}{3} = \left. \frac{2(lnx)^{\frac{3}{2}}}{3} \right|_{1}^{e} = \frac{2(lne)^{\frac{3}{2}}}{3} - \frac{2(ln1)^{\frac{3}{2}}}{3} = \frac{2}{3} - 0 = \frac{2}{3}$$

$$\int \frac{1}{1+x^2}dx \qquad \int \frac{x}{1+x^2}dx \qquad \int \frac{x}{(1+x^2)^7}dx \qquad \int \frac{x+11}{1+x^2}dx$$

$$tan^{-1}x + C \qquad \frac{1}{2}\ln(1+x^2) + C \qquad \frac{(1+x^2)^{-6}}{-12} + C \qquad \int \frac{x}{1+x^2}dx + \int \frac{11}{1+x^2}dx$$

$$\frac{1}{2}\ln(1+x^2) + 11tan^{-1}x + C$$

This is tricky.

$$\int \frac{2x+5}{x^2+4x+5}dx = \int \frac{2x+4+1}{x^2+4x+5}dx = \int \frac{2x+4}{x^2+4x+5}dx + \int \frac{1}{x^2+4x+5}$$

$$\int \frac{2x+4}{x^2+4x+5}dx = \ln(\,x^2+4x+5)$$

$$\int \frac{1}{x^2+4x+5}dx = \int \frac{1}{x^2+4x+4+1}dx = \int \frac{1}{(x+2)^2+1}dx = tan^{-1}(x+2)$$

$$\ln(x^2+4x+5) + tan^{-1}(x+2) + C$$