Calculus 7.1 Slope Fields

If you are given the derivative of a function, you can find the function by integrating if it is an easily integrated function. If it is not, you can use an approximation to graph the equation.

One approximation is a slope field. Graph the derivative of the function.

$\frac{dy}{dx} = x$

. . . . . Connect the lines and

. . . . . what shape do you see?

. . . . .

. . . . .

. . . . .

$\frac{dy}{dx} = x^2$

. . . . .

. . . . .

. . . . .

. . . . .

. . . . .

$\frac{dy}{dx} = x + y$

. . . . .

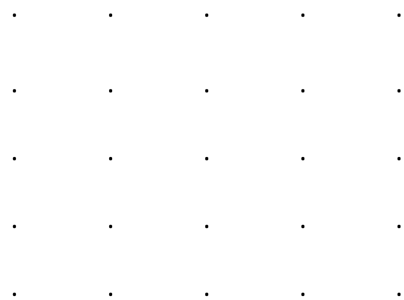
. . . . .

. . . . .

. . . . .

. . . . .

$\frac{dy}{dx} = xy$



Another way to approximate the curve given the derivative is called the Euler Method.

Use the chart below to create points that are close to a given point to create a new point.

If $\frac{dy}{dx} = x$ and you know $f(0) = 2$. Use Euler's method and increments of $\Delta x = .5$ to approximate $f(2)$.

(x, y)	$\frac{dy}{dx}$	Δx	$\Delta y = \frac{dy}{dx}(\Delta x)$	$(x + \Delta x, y + \Delta y)$
$(0, 2)$	0	$.5$	0	$(.5, 2)$
$(.5, 2)$	$.5$	$.5$	$.25$	$(1, 2.25)$
$(1, 2.25)$	1	$.5$	$.5$	$(1.5, 2.75)$
$(1.5, 2.75)$	1.5	$.5$	$.75$	$(2, 3.5)$

How close is this approximation?

Look at the actual value of $f(2)$.

$\int_0^2 x\,dx = f(2) - f(0)$ $\quad \frac{x^2}{2}\Big|_0^2 = \frac{4}{2} - \frac{0}{2} = 2$

$2 = f(2) - 2$ $\quad 4 = f(2)$

Our approximation (3.5) is .5 away from the actual value of 4.

How could we have made the approximation closer?

If we had used a smaller Δx.

Find the general solution to the exact differential equation.

$$\frac{dy}{dx} = secxtanx - e^x \qquad dy = (secxtanx - e^x)dx$$

$$\int dy = \int (secxtanx - e^x)dx \qquad y = secx - e^x + C$$

$$\frac{dy}{dx} = \frac{1}{x} - \frac{1}{x^2}$$

$$\int dy = \int (\frac{1}{x} - \frac{1}{x^2})dx \qquad y = lnx + \frac{1}{x} + C$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{x}}$$

$$\int dy = \int \left(\frac{1}{\sqrt{1-x^2}} - \frac{1}{\sqrt{x}}\right) dx \qquad y = sin^{-1}x - 2\sqrt{x} + C$$

$$\frac{dy}{dx} = (cosx)e^{sinx}$$

$$\int dy = \int (cosx)e^{sinx} dx \qquad y = e^{sinx} + C$$

Solve the initial value problem explicitly.

$\frac{dy}{dx} = 2e^x - cosx$ and $y = 3$ when $x = 0$

$$\int dy = \int (2e^x - cosx)dx \qquad y = 2e^x - sinx + C$$

$3 = 2e^0 - \sin(0) + C \qquad 3 = 2 + C \qquad 1 = C \qquad y = 2e^x - sinx + 1$

$\frac{dy}{dx} = x(3x - 2)$ and $y = 0$ when $x = 1$

Foil it out first and then integrate it.

$\frac{dy}{dx} = 3x^2 - 2x$ $\int dy = \int (3x^2 - 2x)dx$ $y = x^3 - x^2 + C$

$0 = 1^3 - 1^2 + C$ $0 = C$ $y = x^3 - x^2$