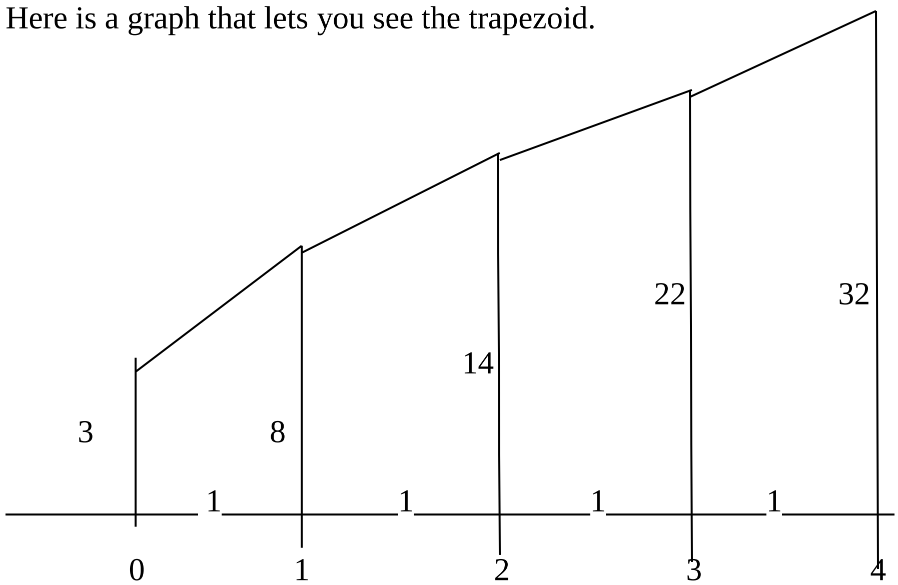
New 6.5 Notes

To approximate the area under the curve of a function, we have used rectangles. Is there another shape that would be a better approximation?

Let's use a trapezoid. Remember the formula $A = \frac{1}{2}h(b_1 + b_2)$.

Let's use the equation $y = x^2 + 3x + 4$ over the interval $[0, 4]$, using four trapezoids.

Here is a graph that lets you see the trapezoid.



Using the equation $y = x^2 + 3x + 4$, you find the bases to the trapezoids.

When $x = 0, y = 4$ $x = 1, y = 8$ $x = 2, y = 14$ $x = 3, y = 22$ $x = 4, y = 32$

You are just adding all of the trapezoids together.

$$A = \frac{1}{2}h(b_1 + b_2) + \frac{1}{2}h(b_2 + b_3) + \frac{1}{2}h(b_3 + b_4) + \frac{1}{2}h(b_4 + b_5)$$

Please notice that you could factor out a $\frac{1}{2}h$ and get

$$A = \frac{1}{2}h\big((b_1 + b_2) + (b_2 + b_3) + (b_3 + b_4) + (b_4 + b_5)\big)$$

which could be simplified to $A = \frac{1}{2}h(b_1 + 2b_2 + 2b_3 + 2b_4 + b_5)$

All of the inside heights (b_2, b_3& b_4) are in two trapezoids, so they are doubled.

The area is $\frac{1}{2}(1)(3 + 2(8) + 2(14) + 2(22) + 32) = \frac{123}{2} = 61.5$.

The actual area is the integral $\int_0^4 (x^2 + 3x + 4)dx = 61.33$ which is very close.

If the x coordinates of the trapezoids are not going up in the same interval, you have to just add the trapezoids separately. You cannot use the formula.

Use the trapezoid rule to approximate the distance traveled using this data. Notice that the speed change is the y axis and the time is the x axis.

Speed change	Time (sec)
Zero to	
30 mph	2.0
40 mph	3.2
50 mph	4.5
60 mph	5.8
70 mph	7.7

$$\frac{1}{2}(2)(0+30)+\frac{1}{2}(1.2)(30+40)+\frac{1}{2}(1.3)(40+50)+\frac{1}{2}(1.3)(50+60)+\frac{1}{2}(1.9)(60+70)=325$$

If the function is concave up, the trapezoids would create an overestimate.

If the function is concave down, the trapezoids would create an underestimate.