New 6.4 Notes

Let $y = \int_0^x t dt$, what is y' ?

$$y = \frac{t^2}{2}\Big|_0^x = \frac{x^2}{2} - 0 = \frac{x^2}{2} \qquad\qquad y' = \frac{2x}{2} = x$$

Let $y = \int_0^x cost dt,$ what is y′ ?

$$y = sint|_0^x = sinx - sin0 = sinx \qquad\qquad y' = cosx$$

Is there any easier way to solve this problem?

Just put the x in for the t in the integrand (the function being integrated).

Let's try the shortcut method.

Let $y = \int_0^x e^t dt$, what is y' ? $y' = e^x$

In essence, we are just taking the derivative of the integral and getting the initial function.

Where else does this happen? If you take the square root of a function that is squared, you should get the original function.

Does this shortcut work all of the time?

Let $y = \int_0^{x^2} 2t dt$, what is $'$? The shortcut says $y' = 2x^2$.

Let's try it to see if it works.

$$y = \int_0^{x^2} 2t dt = t^2|_0^{x^2} = (x^2)^2 - 0 = x^4 \qquad\qquad y' = 4x^3$$

What are we missing? 2x Can I find the 2x?

That is the derivative of x^2.

Can we create a formula for any function?

Let $y = \int_0^{f(x)} g(t) dt,$ $y' = f'(x)g(f(x))$

Does the lower value have to be a 0?

$$y = \int_4^x e^t dt \qquad y = e^t|_4^x \quad y = e^x - e^4 \quad y' = e^x - 0 = e^x$$

The bottom value goes away if it is a constant.

What if the bottom value is not a constant?

$$y = \int_x^{x^2} \frac{1}{t} dt = lnt|_x^{x^2} = lnx^2 - lnx \qquad y' = \frac{2x}{x^2} - \frac{1}{x}$$

This is our pattern but we would reduce the x's to $\frac{2}{x} - \frac{1}{x} = \frac{1}{x}$.

Let's create a pattern for a function in the top value and bottom value.

If $y = \int_{g(x)}^{f(x)} h(t)dt$, $\qquad y' = f'(x)h\big(f(x)\big) - g'(x)h(g(x))$

This is called the Fundamental Theorem of Calculus Part One.

Yeah, this is important.

You will need to know this on the test, I promise.

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Oh yes, there is a Part 2.

Fundamental Theorem of Calculus Part Two

If f is continuous at every point of [a, b], and if F is any antiderivative of f on [a, b],

$$\int_a^b f(x)dx = F(b) - F(a)$$

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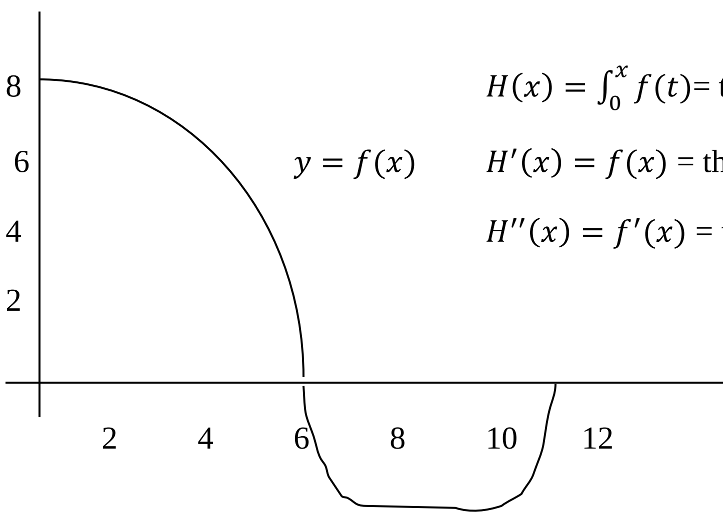
Let $H(x) = \int_0^x f(t)dt$, where f is the continuous function with domain [0, 12] graphed here.



$H(x) = \int_0^x f(t)$= the area under the curve from 0 to x

$H'(x) = f(x)$ = the y coordinate

$H''(x) = f'(x)$ = the slope of the curve

Find $H(0)$. $H(0)$ is the area from 0 to 0 which is 0.

On what interval is H increasing? Explain. H is increasing when $H' = f(x) > 0$. When the y coordinate is greater than 0, the function H is increasing. (0, 6)

On what interval is H concave up? Explain. H is concave up when $H'' = f' or\ the\ slope\ of\ the\ curve$ is greater than 0. This occurs (10, 11).

Is $H(12)$ positive or negative? This is the area from 0 to 12. There is more positive area than negative, so $H(12)$ is positive.

Where does H achieve its maximum value? Explain. H has a maximum when $H'($ $f(x)$ which is the y-value) goes from positive to negative or at an endpoint. Positive to negative occurs when $x = 6$. The endpoints are $t = 0$ or $t = 11$. $H(0) = 0$ and $H(11)$ is less than $H(6)$ because there is negative area from 6 to 11. The maximum is at $t = 6$.

Where does H achieve its minimum value? Explain. H has a minimum value when $H'(f(x)$ which is y-value) goes from negative to positive or at an endpoint. $H'(x)$ never goes from negative to positive. $H(0) = 0$ and $H(11) > 0$ because the area above the x-axis is larger than the area below the x-axis.

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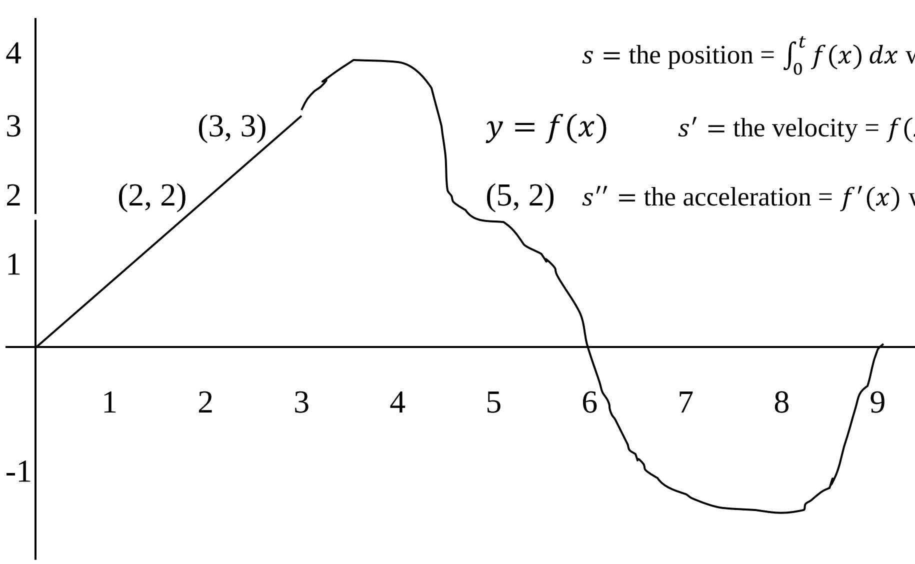
f is the differentiable function whose graph is shown in the given figure. The position at t (sec) of a particle moving along a coordinate axis is $s = \int_0^t f(x)dx$ meters. Use the graph to answer the questions. Give reasons for your answers.



s = the position = $\int_0^t f(x)\,dx$ which is the area from 0 to x.

s' = the velocity = $f(x)$ which is the y coordinate

s'' = the acceleration = $f'(x)$ which is the slope of the curve

What is the particle's velocity at $t = 5$? The velocity is $s' = f(x) = 2$ at $t = 5$.

Is the acceleration of the particle at time $t = 5$ positive or negative? The acceleration is s'' = the slope of the curve. The slope of the curve at $t = 5$ is negative so the acceleration is negative.

What is the particle's position at time $t = 3$? The position, which is the area under the curve can be found by finding the area of the triangle. $A = \frac{1}{2}bh = \frac{1}{2}(3)(3) = 4.5$.

At what time during the first 9 seconds does s have its largest value? s has a maximum when s' (which is the y coordinate) goes from positive to negative or at an endpoint. s' goes from pos to neg at 6. $s(0) = 0$ and $s(9)$ is less than $s(6)$ because it has negative area after 6.

Approximately when is the acceleration zero? The acceleration is s'', which is the slope of the curve. The slope of the curve is 0 at $t = 4, 8$.

When is the particle moving toward the origin? Away from the origin?

The particle is moving in a positive direction (away from the origin) from 0 to 6 seconds. From 6 to 9 seconds, the particle is moving in a negative direction (back towards the origin).

On which side of the origin does the particle lie at time $t = 9$? At $t = 9$, there is more positive area than negative area, so the particle is to the right of the origin