New 6.3 Notes

$$\int_a^b f(x)dx = -\int_b^a f(x)dx$$

$$\int_a^a f(x)dx = 0$$

$$\int_a^b 15f(x)dx = 15\int_a^b f(x)dx$$

$$\int_a^b (f(x) + g(x))dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$

$$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$$

$\int_1^7 f(x)dx = 10$ $\int_7^{11} f(x)dx = -11$ $\int_1^7 g(x)dx = 7$

$\int_1^1 f(x)dx = 0$ $\int_7^1 f(x)dx = -10$ $\int_1^7 6f(x)dx = 60$

$\int_1^7 4f(x)dx - \int_1^7 11g(x)dx = 40 - 77 = -37$

If $y = 4x$, what is the average value (y-value) between x = 2 & x = 8?

When x = 2, y = 8 and when x = 8, y = 32.

The average value is $\frac{8+32}{2} = \frac{40}{2} = 20$.

Here is another way to find the average value :

If you graph $y = 4x$, the area under the curve is

$\int_2^8 4xdx =$ a rectangle with width $(8-2)$ with height $f(c)$ which is the average height of the function between x = 2 & x = 8.

$(8-2)f(c) = \int_2^8 4xdx$

$$f(c) = \frac{1}{8-2}\int_2^8 4xdx = \frac{1}{6}\int_2^8 4xdx = \frac{1}{6}(120) = 20$$

The Mean Value Theorem

If f is continuous on $[a, b]$, then at some point c in $[a, b]$,

$f(c) = \frac{1}{b-a}\int_a^b f(x)dx.$

Find the mean value of the function $f(x) = x^2 - 4$ over [1, 4]. Where does the function f actually take on that value in the interval?

$$f(c) = \frac{1}{4-1}\int_1^4 (x^2 - 4)dx = 9$$

$x^2 - 4 = 9 \quad x^2 = 13 \quad x = \pm\sqrt{13} \quad x = \sqrt{13}$

Evaluate the integral using anti-derivatives.

$$\int_{0}^{\pi} sinxdx = -cosx\Big|_{0}^{\pi} = -cos\pi + cos0 = 1 + 1 = 2$$

$$\int_{1}^{3} 5dx = 5x\Big|_{1}^{3} = 5(3) - 5(1) = 15 - 5 = 10$$

$$\int_{1}^{e} \frac{1}{x}dx = lnx\Big|_{1}^{e} = lne - ln1 = 1 - 0 = 1$$

$$\int_{0}^{\frac{\pi}{4}} sec^2xdx = tanx\Big|_{0}^{\frac{\pi}{4}} = tan\frac{\pi}{4} - tan0 = 1 - 0 = 1$$

$$\int_{-3}^{1} -\frac{3}{x^2}dx = \int_{-3}^{1} -3x^{-2}dx = 3x^{-1}\Big|_{-3}^{1} = \frac{3}{x}\Big|_{-3}^{1} = \frac{3}{1} - \frac{3}{-3} = 3 + 1 = 4$$