### 6.1 Notes

How far would you travel if your velocity was 50 mph for 6 hours? 300 miles
If you create a velocity graph with a constant velocity of 50 and your time equals 6 , how could you find the distance traveled? Find the area of the rectangle. $50(6)=300$


What if the velocity is not consistent? Could you still find the area under the curve?
Split up the area under the curve into rectangles.
Look at the curve $y=x^{2}$ over the interval $[0,4]$.


You could make the area into 4 rectangles that has a base of 1 . The height is changing from 0 to 3 .

| $x$ | 0 | 1 | 2 | 3 |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $f(x)=x^{2}$ | Height | 0 | 1 | 4 | 9 |
| Base |  | 1 | 1 | 1 | 1 |
| Area |  | 0 | 1 | 4 | 9 |

If you add the areas together you get 14.
Is this area smaller or larger than the true area? Smaller.

This is called the left-hand endpoint rectangular approximation method or LRAM.
We started at the utmost left y coordinate for the height.

Look at the curve $y=x^{2}$ over the interval $[0,4]$.


You could make the area into 4 rectangles that has a base of 1 . The height is changing from 1 to 4 .

| $x$ | 4 | 3 | 2 | 1 |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $f(x)=x^{2}$ | Height | 16 | 9 | 4 | 1 |
| Base |  | 1 | 1 | 1 | 1 |
| Area |  | 16 | 9 | 4 | 1 |

If you add the areas together you get 30 .
Is this area smaller or larger than the true area?
Larger.

This is called the right-hand endpoint rectangular approximation method or RRAM.
We started at the utmost right y coordinate.

Look at the curve $y=x^{2}$ over the interval $[0,4]$.


You could make the area into 4 rectangles that has a base of 1 . The height is changing from .5 to 3.5 .

| $x$ |  | .5 | 1.5 | 2.5 | 3.5 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $f(x)=x^{2}$ | Height | .25 | 2.25 | 6.25 | 12.25 |
| Base |  | 1 | 1 | 1 | 1 |
| Area |  | .25 | 2.25 | 6.25 | 12.25 |

If you add the areas together you get 21.
Is this area smaller or larger than the true area?
Smaller.

This is called the midpoint rectangular approximation method or LRAM.
We used the midpoints for the y coordinates.

Can we get the real answer?
Look at the equation $y=2 x$. What is the area under the curve $[0,4]$ ?


We can find the area under the curve because it is a triangle with base 4 , height 8 , the area is

$$
\frac{1}{2}(4)(8)=16
$$

Is there another way to find the actual area?
We take the antiderivative of $y=2 x$, you get $x^{2}$.
If you plug in 4 , you get 16 . If you plug in 0 , you get $0.16-0=16$.

Go back to $y=x^{2}$, the antiderivative is $\frac{x^{3}}{3}$.
If you plug in 4 , you get $\frac{64}{3}$. If you plug in 0 , you get $0 . \frac{64}{3}-0=\frac{64}{3}$.

Go to your calculator. Put in $y=x^{2}$. Second trace. $\int f(x) d x$.
Lower boundary 0. Enter. Upper boundary 4. Enter. 21.333

Approximate the area of the region enclosed between the graph of $f(x)=\frac{1}{x}$ and the x -axis over the region [1,5] using LRAM, RRAM, MRAM and 4 rectangles. Find the actual area using the calculator.

| $x$ | 1 | 2 | 3 | 4 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $f(x)=$ height | 1 | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{4}$ |  |  |  |  |  |
| Width | 1 | 1 | 1 | 1 |  |  |  |  |  |
| Area | 1 | $\frac{1}{2}$ | $\frac{1}{3}$ | $\overline{4}$ |  |  |  |  |  |
| $\text { LRAM }=\text { Addin }$ |  |  |  |  |  |  |  |  |  |
| $x$ | 2 | 3 | 4 | 5 | 1 | 2 | 3 | 4 | 5 |
| $f(x)=$ height | $\frac{1}{2}$ | $\frac{1}{3}$ | $\frac{1}{4}$ | $\frac{1}{5}$ |  |  |  |  |  |
| Width | 1 | 1 | 1 | 1 |  |  |  |  |  |
| Area | $\frac{1}{2}$ | $\underline{1}$ | $\frac{1}{4}$ | $\frac{1}{5}$ |  |  |  |  |  |

RRAM $=$ adding the areas $=\frac{77}{60}=1 \frac{17}{60}=1.283$

| $x$ | 1.5 | 2.5 | 3.5 | 4.5 |
| :--- | :---: | :---: | :---: | :---: |
| $f(x)=$ height | $\frac{2}{3}$ | $\frac{2}{5}$ | $\frac{2}{7}$ | $\frac{2}{9}$ |
| Width | 1 | 1 | 1 | 1 |
| Area | $\frac{2}{3}$ | $\frac{2}{5}$ | $\frac{2}{7}$ | $\frac{2}{9}$ |

RRAM $=$ adding the areas $=\frac{496}{315}=1 \frac{181}{315}=1.575$

Go to your calculator. Put in $y=\frac{1}{x}$. Second trace. $\int f(x) d x$.
Lower boundary 1. Enter. Upper boundary 5. Enter. 1.609

