New Calculus 5.6 Related Rates

I have the ability to change every attitude in this room for the time of my class period.

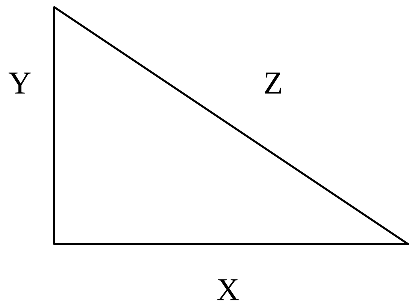
I can make you have the best day or I can make you have the worst day.

After you have been in my class, you will affect every student that you come in contact with in either a good way or a bad way.

If I inspire you, you might want to inspire others.

If I destroy your self esteem, you might want to do the same thing to others.

One day a man changed my family’s life.



Let this right triangle have a base x, height y and diagonal z.

If I change the x value, what has to change? Either the y or the z.

If I change the y value, what has to change? Either the x or the z.

If I change the z value, what has to change? Either the y or x.

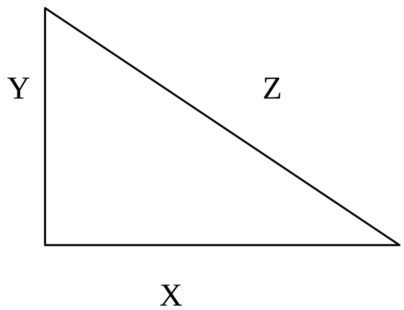
This is a related rate problem. More than one variable can be changing at one time.

When this happens, every variable that is changing has to be dealt with implicitly.

In this picture, the relationship $x^2 + y^2 = z^2$ is evident.

The derivative of $x^2 + y^2 = z^2$ is $2xx' + 2yy' = 2zz'$ where all of the variables are changing.

If I make this a "falling ladder" problem with the ladder z, the wall y and the x being the distance the bottom of the ladder is from the wall, which variable is not changing?



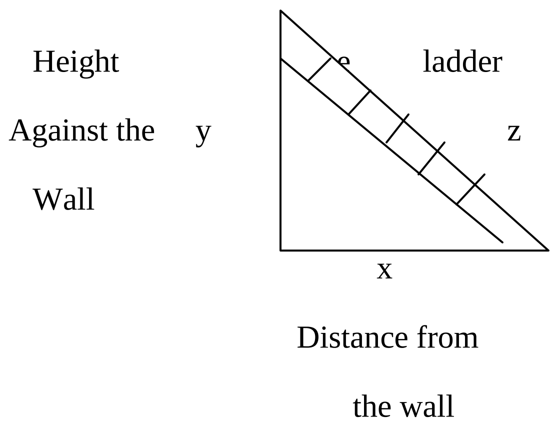
If in the above picture, $x = 8\ ft,\ y = 15ft,\ \frac{dx}{dt} = 3\frac{ft}{sec}$ & $\frac{dy}{dt} = -1ft/sec$.

What is $\frac{dz}{dt}$?

Use the Pythagorean Theorem two times, once for the distance and once for the changes.

Distance	Changes
$x^2 + y^2 = z^2$	$x^2 + y^2 = z^2$
$8^2 + 15^2 = z^2$	$2xx' + 2yy' = 2zz'$
$64 + 225 = z^2$	$2(8)(3) + 2(15)(-1) = 2(17)z'$
$289 = z^2$	
$17 = z$ Use this over here.	$48 - 30 = 34z'$ $\quad z' = \frac{18}{34} = \frac{9}{17}ft/sec$

The Sliding Ladder Problem



The ladder is sliding down the wall at a rate of $5\,\frac{ft}{sec}$. When the base of the 25 ft ladder is 7 ft from the wall, how fast is the ladder moving away from the wall?

Use the Pythagorean Theorem two times, once for the distance and once for the changes.

Distance	Changes
$x^2 + y^2 = z^2$	$x^2 + y^2 = z^2$
$7^2 + y^2 = 25^2$	$2xx' + 2yy' = 2zz'$ $z' = 0$ as the ladder does not change size
$y = 24$	$2(7)x' + 2(24)(-5) = 2(25)(0)$
	$14x' - 240 = 0$
	$14x' = 240$
	$x' = \frac{240}{14} = \frac{120}{7}$

How fast is the angle between the ladder and the ground changing at the same time?

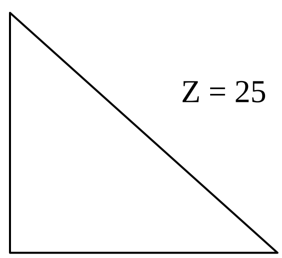
We need to create a relationship between the angles and the sides of the triangle.

We can use any trig relationship but we will find that some are easier than others.

$$sin\theta = \frac{y}{z} \qquad cos\theta = \frac{x}{z} \qquad tan\theta = \frac{y}{x}$$

Which variable is not changing? z

We want the z in the equation so we do not have a z'.



$$x' = \frac{120}{7}$$

$$sin\theta = \frac{y}{25}$$

$$25sin\theta = y$$

$$25\theta' cos\theta = y'$$

$$25\theta'\left(\frac{7}{25}\right) = -5$$

$$7\theta' = -5$$

$$\theta' = \frac{-5}{7}$$

$$cos\theta = \frac{x}{25}$$

$$25cos\theta = x$$

$$-25\theta' sin\theta = x'$$

$$-25\theta'\left(\frac{24}{25}\right) = \frac{120}{7}$$

$$-24\theta' = \frac{120}{7}$$

$$\theta' = -\frac{120}{7(24)} = -\frac{5}{7}$$

$$tan\theta = \frac{y}{x}$$

$$\theta' sec^2\theta = \frac{xy' - x'y}{x^2}$$

$$\theta'\left(\frac{25}{7}\right)^2 = \frac{7(-5) - \frac{120}{7}(24)}{49}$$

$$\theta' = \frac{49}{625}\left(\frac{-35 - \frac{120}{7}(24)}{49}\right)$$

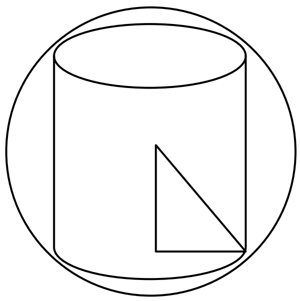
$$\theta' = \frac{-35 - \frac{120}{7}(24)}{625}$$

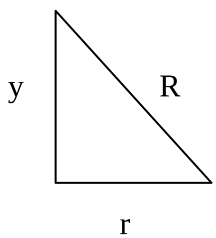
$$\theta' = \frac{-35(7) - 120(24)}{7(625)} =$$

$$\theta' = -\frac{-49 - 24(24)}{7(125)} = -\frac{625}{7(125)}$$

$$\theta' = -\frac{5}{7}$$

Find the volume of the largest cylinder that can be inscribed in a sphere of radius 6.





y = ½ of the height of the cylinder

R = the radius of the sphere

r = the radius of the cylinder

$y^2 + r^2 = R^2$

$y^2 + r^2 = 36$

$r^2 = 36 - y^2$

$$V = \pi r^2 h = \pi(36 - y^2)(2y)$$

$$V = \pi(72y - 2y^3)$$

$$V' = \pi(72 - 6y^2)$$

$$0 = \pi(72 - 6y^2)$$

$$0 = 72 - 6y^2$$

$$6y^2 = 72$$

$$y^2 = 12$$

$$y = 2\sqrt{3}$$

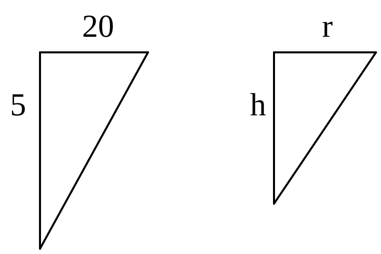
$$h = 2y = 4\sqrt{3} \qquad r = 36 - (2\sqrt{3})^2 = 24$$

$$V = \pi r^2 h = 96\sqrt{3}$$

Conical Reservoir

Water runs into a conical tank at a rate of $16\pi m^3/min$ The tank stands point down and has a base radius of 20 m and height 5 m.

a) How fast is the water level falling when the water is 2 meters?
b) How fast is the radius of the water's surface changing at that moment?



$\frac{20}{5} = \frac{r}{h}$ $\quad 4 = \frac{r}{h}$ $\quad 4h = r$ $\quad 4h' = r'$

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi(4h)^2 h = \frac{16\pi h^3}{3}$$

$$V' = \frac{3(16\pi h^2 h')}{3} = 16\pi h^2 h'$$

$$-16\pi = 16\pi(2)^2 h'$$

$$-16\pi = 64\pi h' \qquad -\frac{1}{4} = h' \qquad -1 = r'$$