New Calculus 5.5 Linearization and Differentials

Differentiable curves are locally linear.

If you zoom in at a point, you can make any curve into a line.

The Linearization Formula

$L(x) = f(a) + f'(a)(x - a)$

What does the formula look like?

Play some Algebra and you get

$L(x) - f(a) = f'(a)(x - a)$ This is point slope?

$\frac{L(x)-f(a)}{x-a} = f'(a)$ This is the slope formula.

Find the linearization of $f(x) = x^3 - 2x + 3$ at $a = 2$ and use it to approximate 2.01.

What is $f(2.01)$? What is the error of our approximation?

$f'(x) = 3x^2 - 2 \quad f'(2) = 10 \quad f(2) = 7$

$L(x) = f(a) + f'(a)(x - a)$

$L(x) = 7 + 10(x - 2)$

$L(2.01) = 7 + 10(2.01 - 2) = 7 + 10(.01) = 7 + .1 = 7.1$

$f(2.01) = 7.100601$

The error is .00601.

Use linearizations to approximate $\sqrt{101}$. What is the actual value and what is the error?

Let $f(x) = \sqrt{x}$ and let $a = 100$ as this is the closest perfect square root to 101.

$f'(x) = \frac{1}{2\sqrt{x}} \quad f'(100) = \frac{1}{2\sqrt{100}} = \frac{1}{20} \quad f(100) = 10$

$L(x) = f(a) + f'(a)(x - a) = 10 + \frac{1}{20}(x - 100)$

$L(101) = 10 + \frac{1}{20}(101 - 100) = 10 + \frac{1}{20} = 10.05$

$f(101) = \sqrt{101} = 10.049875$

The error is .000125.

Newton's Method

The objective of Newton's Method is to find the zero of an equation by using tangent lines.

We will do it without a calculator first to show you how to do it. Then you will realize how easy the calculator method is.

Let $f(x) = x^3 + 3x + 1$.

You guess at what you think the zero might be. Try $x = -1$.

Find the tangent line to $f(x)$ at $x = -1$.

You need a point and a slope.

When $x = -1, y = \ -3 \ and \ f'(x) = 3x^2 + 3 \quad f'(-1) = 6$

$y + 3 = 6(x + 1)$ Set $y = 0$ and solve for x.

$3 = 6x + 6 \quad -3 = 6x \quad -\frac{1}{2} = x$

Now do the same thing using $x = -\frac{1}{2}$.

When $x = -.5 \quad y = \ -14.25 \ and \ f'(x) = 3x^2 + 3 \quad f'(-.5) = 3.75$

$y + 14.25 = 3.75(x + .5)$ Set $y = 0$ and solve for x.

$14.25 = 3.75(x + .5)$

$3.8 = x + .5$

$3.3 = x$

Use the calculator to do this problem.

Enter the function as y_1 and the derivative as y_2. $y_1 = x^3 + 3x + 1$

$y_2 = 3x^2 + 3$

Quit

Store the initial guess as x -1 sto→ x Enter

X – Vars (Y-Vars) Function Y_1 Enter / ÷ Vars (Y-Vars) Function Y_2 Enter Sto→ x

Keep pressing the enter key until the digits are repeating themselves.

This is your answer.