Review for 5.4 – 5.6

The sum of two non-negative number is 16. Find the numbers if

a) the sum of their squares is as large as possible.
b) the sum of their squares is as small as possible.

Let $x =$ the first number and $16 - x =$ the second number.

$f(x) = x^2 + (16 - x)^2$

$f'(x) = 2x - 2(16 - x) = 4x - 32$

$0 = 4x - 32 \quad x = 8$

x	0	8	16
$f'(x)$	-	0	+
I, D, LMAX or LMIN	D	L MIN	I
$f(x)$	256	128	256
ABS MAX ABS MIN	ABS MAX	ABS MIN	ABS MAX

c) one number plus the square root of the other is as small as possible.

$f(x) = \sqrt{x} + 16 - x$

$f'(x) = \frac{1}{2\sqrt{x}} - 1 = 0 \qquad \frac{1}{2\sqrt{x}} = 1 \qquad 1 = 2\sqrt{x} \qquad \frac{1}{2} = \sqrt{x} \qquad \frac{1}{4} = x$

x	0	$\frac{1}{9}$	$\frac{1}{4}$	16
$f'(x)$	Und	+	0	-
I, D, LMAX or LMIN			L MAX	D
$f(x)$	16		$16\frac{1}{4}$	4
ABS MAX ABS MIN	L MIN		ABS MAX	ABS MIN

When the numbers are $\frac{1}{4}$ & $15\frac{3}{4}$ the value is the largest.

When the numbers are 16 & 0, the value is the smallest.

The area of a rectangle is 25. Find the dimensions of the smallest perimeter that can be constructed.

Let $x = length$ and $\frac{25}{x} = width$.

The perimeter $f(x) = 2x + 2\left(\frac{25}{x}\right) = 2x + 50x^{-1}$

$f'(x) = 2 - 50x^{-2} = 0 \quad \frac{50}{x^2} = 2 \quad 50 = 2x^2 \quad x^2 = 25 \quad x = \pm 5 \rightarrow x = 5$

x	0	1	5	100
$f'(x)$	Und	-	0	+
I, D, LMAX or LMIN	X	D	L MIN	I
$f(x)$	X	52	20	200.5
ABS MAX ABS MIN	X		ABS MIN	

When the sides are 5 & 5, the perimeter is the smallest.

Find the dimensions that would create the smallest area inscribed under the curve

$f(x) = 3cos2x$ with the x-axis at its base.

The area $A = 2x(3cos2x) = 6xcos2x.$

$A' = 6(cos2x - 2xsinx)$

$6(cos2x - 2xsinx) = 0\ at\ x = .43$

The dimensions are 2(.43) by 3cos(.86) = 1.68

Find the dimensions of the lightest square based, open-top, rectangular box that can be made with a volume of 4000 ft^3.

Let $x =$ the side of the square base.

Let $h =$ the height of the box.

$V = x^2 h \qquad 4000 = x^2 h \qquad \frac{4000}{x^2} = h$

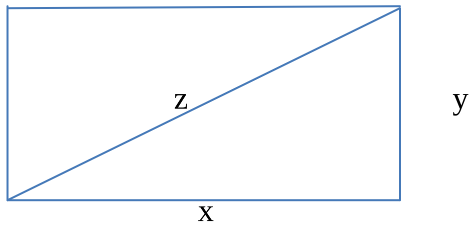
Surface area = $4 sides + the\ base = 4x\left(\frac{4000}{x^2}\right) + x^2 = SA$

$SA = \frac{16000}{x} + x^2 = 16000x^{-1} + x^2$

$SA' = -16000x^{-2} + 2x = 0$

$\frac{16{,}000}{x^2} = 2x \qquad 16{,}000 = 2x^3 \qquad 8{,}000 = x^3 \qquad x = 20$

The dimensions are 20 by 20 by 10.



$x = 24, z = 26, x' = 2, y' = 3$

What is z'?

$x^2 + y^2 = z^2$

$24^2 + y^2 = 26^2$

$y = 10$

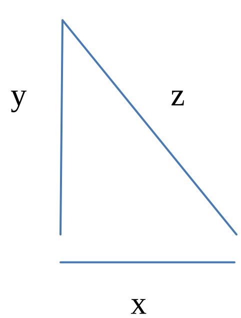
$x^2 + y^2 = z^2$

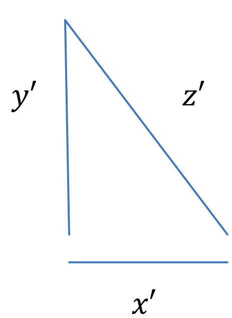
$2xx' + 2yy' = 2zz'$

$2(24)2 + 2(10)(3) = 2(26)z'$

$96 + 60 = 52z' \qquad z' = \frac{156}{52} = 3$

A 30 foot ladder is 24 feet from the wall and falling at a rate of 3 ft/sec. How fast is it moving away from the wall? What is the rate of the angle between the ladder and the ground?





$x^2 + y^2 = z^2$ $x^2 + y^2 = z^2$

$24^2 + y^2 = 30^2$ $2xx' + 2yy' = 2zz'$

$y = 18$ $2(24)(x') + 2(18)(-3) = 0$

$48x' - 108 = 0$

$x' = \frac{108}{48} = 2.25$

$sin\theta = \frac{y}{z}$ $sin\theta = \frac{y}{30}$ $30sin\theta = y$ $30cos\theta\theta' = y'$

$30\left(\frac{24}{30}\right)\theta' = -3$ $24\vartheta' = -3$ $\vartheta' = -\frac{1}{8}$ rad/sec

A conical reservoir with vertex down is leaking water at a rate of $24\pi\ \frac{m^3}{hour}$. If the initial height is 12 m and the radius is 6 m, find how fast the radius and height are changing when the height is 5.

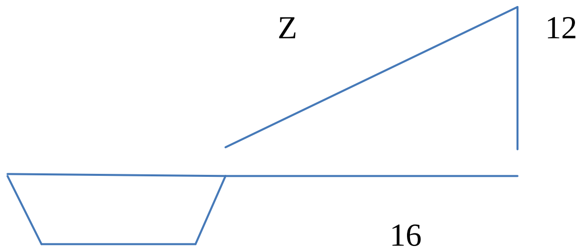
$h = 2r$ If the height is 5, the radius is 5/2

$$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2 (2r) = \frac{2\pi r^3}{3}$$

$V' = 2\pi r^2 r'$ $-24\pi = 2\pi(\frac{5}{2})^2 r'$ $-24\pi = \frac{2\pi r'(25)}{4}$

$-96\pi = 50\pi r'$ $r' = -\frac{96}{50} = -\frac{48}{25}$

A dinghy (small boat) is being pulled into shore with a rope from a 12 foot dock at a rate of 3 ft/sec. When the boat is 16 feet from the dock, how fast is the boat approaching the dock?



$$x^2 + y^2 = z^2 \qquad z = 20$$

$$2xx' + 2yy' = 2zz'$$

$$2(16)x' + 2(12)(0) = 2(20)(-3)$$

$$32x' = -120 \qquad x' = -\frac{120}{32} = -\frac{15}{4}$$

The boat is approaching the dock at $\frac{15}{4} ft/sec$.