New Calculus 5.3 Connecting f'& f'' with the Graph of f

The First Derivative Test

At a critical point c, if f' changes from $+ \ to \ -$, then f has a local maximum value at c.

At a critical point c, if f' changes from $- \ to \ +$, then f has a local minimum value at c.

Let's try it.

$f(x) = x^3 - 12x + 7$

Use THE CHART model from the last section.

$f'(x) = 3x^2 - 12 = 3(x^2 - 4)$

$f'(x) = 0$ at $x = \pm 2$

These are the critical values.

Pick values to the left and right of the critical values. (There are no boundaries to this equation.)

Find the derivatives at these points.

See if you have the above patterns.

x	-3	-2	0	2	3
$f'(x)$	+	0	-	0	+
I or D	I		D		I
Local maximum or minimum		Local Maximum		Local Minimum	
$f(x)$		11		-9	
Absolute maximum or minimum	No		Boundaries		

$g(x) = (x^2 - 3)e^x$

Use THE CHART method.

1 Find the derivative. $2x(e^x) + (x^2 - 3)e^x = e^x(x^2 + 2x - 3)$

2 Set the derivative = 0. $e^x \neq 0$ so let $x^2 + 2x - 3 = 0$ $(x + 3)(x - 1) = 0$

$x = -3, 1$

3 Create THE CHART

x	-4	-3	0	1	2
$f'(x)$	+	0	-	0	+
I or D	I		D		I
Local maximum or minimum		Local Maximum		Local Minimum	
$f(x)$	Going to 0	$6e^{-3}$		$-2e$	
Absolute maximum or minimum				Absolute minimum	

Concavity

Concavity is determined by the second derivative of a function.

Let's look at different options.

$y = -x^2$ What does this graph look like?

$y = -x^2$ $y' = -2x$ $y'' = -2$

This is an example of concave down for the entire graph.

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$y = x^2$ What does this graph look like?

$y = x^2$ $y' = 2x$ $y'' = 2$

This is an example of concave up for the entire graph.

The Concavity Rule

If $y'' > 0$, the function is concave up. Hooking up.

If $y'' < 0$, the function is concave down. Hooking Down

Some graphs have both types of concavity in it.

$y = x^3$ What does the graph look like?

$y = x^3$ $\quad y' = 3x^2$ $\quad y'' = 6x$

From $(-\infty, 0)$, the graph is concave down and from $(0, \infty)$, the graph is concave up.

At 0, the concavity changes.

Point of Inflection

A point of inflection is where the concavity changes.

Special note:

If there is a point of inflection, the second derivative at that point will be 0 or undefined.

If the second derivative is 0 or undefined at a point, it does not have to be a point of inflection.

Find the point of inflection for $y = tan^{-1}x$.

$y' = \frac{1}{1+x^2} = (1+x^2)^{-1}$

$y'' = -(1+x^2)^{-2}(2x) = \frac{-2x}{(1+x^2)^2}$

$y'' = 0$ when $-2x = 0$ $\quad x = 0$

y'' is undefined when $(1+x^2)^2 = 0$ $\quad 1 + x^2 = 0$ $\quad -1 = x^2$ $\quad x$ is imaginary.

Creating a small chart will help us.

x	-1	0	1
$f''(x)$	+	0	-
Concave Up or Down	Up		Down
POI		POI	

If $y = 4x^2 + 9x - 2$

$y' = 8x$ At $x = 0, y' = 0.$

$y'' = 8$

There is a local minimum at $x = 0$.

The Second Derivative Test

When the first derivative equals 0 and $y'' > 0$, the point is a local minimum.

When the first derivative equals 0 and $y'' < 0$, the point is a local minimum.