New 5.2 Calculus Mean Value Theorem

Graph $y = \frac{1}{4}x^2$.

Find the slope between $(0,0)\&(4,4)$. $\frac{4-0}{4-0} = \frac{4}{4} = 1$.

The Mean Value Theorem says that there must be a point on the curve $y = \frac{1}{4}x^2$that has a tangent line whose slope is 1.

To find the slope of the tangent line, you need to take the derivative.

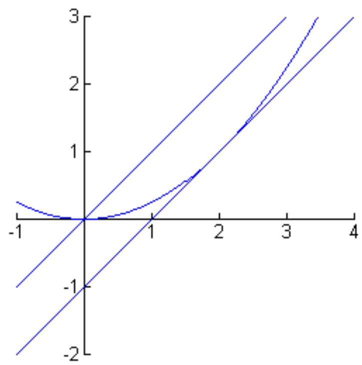
$y' = \frac{x}{2}$

To find the point where the tangent line is parallel to the slope you make the

Derivative at one point equals the slope between two points.

$\frac{x}{2} = 1 \quad x = 2$

Graph it and check to see if it works.



The Mean Value Theorem says that if you take two points and draw the line between them, there has to be a point whose tangent line is parallel to the line between the two points.

$$f'(c) = \frac{y_2 - y_1}{x_2 - x_1}$$

The derivative at c equals the slope between (x_1, y_1) & (x_2, y_2).

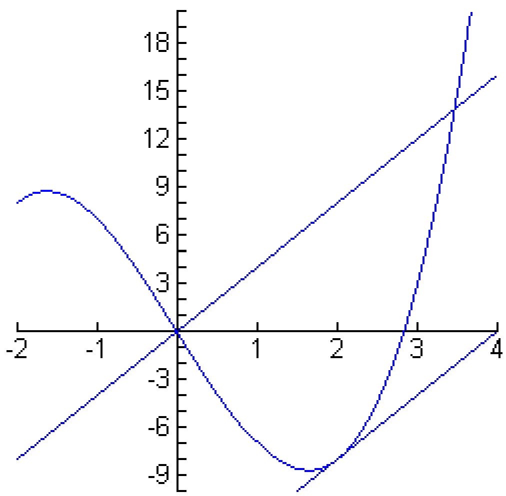
If $y = x^3 - 8x$, find the c value that satisfies the Mean Value Theorem over the interval [0, 4].

$f(0) = 0 \quad f(4) = 16 \qquad \frac{16-0}{4-0} = 4$

$y' = 3x^2 - 8$

$3x^2 - 8 = 4 \quad 3x^2 = 12 \qquad x^2 = 4 \quad x = \pm 2$

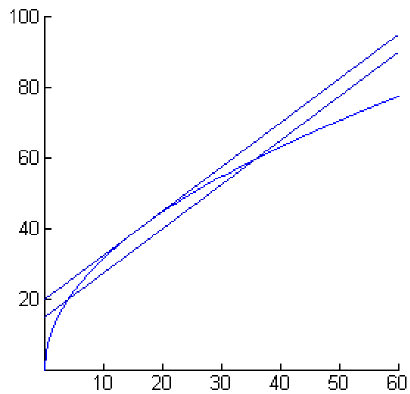
Staying within the boundaries of [0, 4] the answer is $x = 2$.



If $= 10\sqrt{x}$, find the c value that satisfies the Mean Value Theorem over the interval [4, 36].

$f(4) = 20 \quad f(36) = 60 \quad m = \frac{60-12}{36-4} = \frac{5}{4}$

$y' = \frac{5}{\sqrt{x}} \qquad \frac{5}{\sqrt{x}} = \frac{5}{4} \qquad 20 = 5\sqrt{x} \quad 4 = \sqrt{x} \quad x = 16$



Increasing and Decreasing Functions

If $f'(x) > 0$, the function is increasing.

If $f'(x) < 0$, the function is decreasing.

THE CHART

Put the x-value boundary endpoints or the x-value domain endpoints in the chart.

Find the derivative.

Find all points where the derivative equals 0 or is undefined.

Make up values to the left and right of the points where the derivative is 0.

Plug in the x values to $f'(x)$ to see if $f'(x) > 0$ or $f'(x) < 0$

Note if the function is increasing or decreasing

Determine local maximum or minimums.

Plug in x values for endpoints and local maximum or minimum to find the absolute max or min.

$f(x) = x^3 + 6x^2 + 9x - 2$ over [-5, 3]

$f'(x) = 3x^2 + 12x + 9$

$0 = 3x^2 + 12x + 9$

$0 = 3(x^2 + 4x + 3) = (x + 3)(x + 1)$

$x = -1, -3$

$f'(-5) = (-5 + 3)(-5 + 1) = (-)(-) = +$

$f'(-2) = (-2 + 3)(-2 + 1) = (+)(-) = -$

$f'(0) = (0 + 3)(0 + 1) = (+)(+) = +$

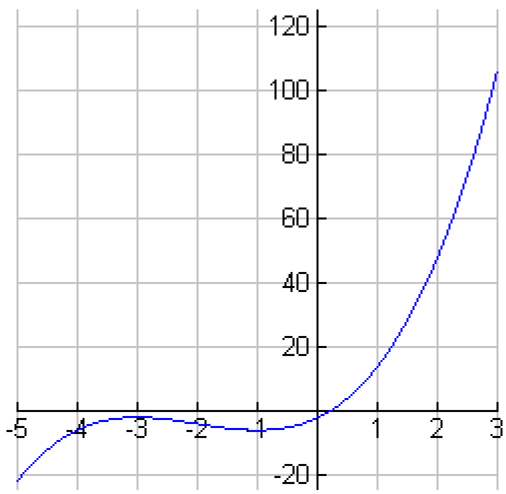
$f(-5) = (-5)^3 + 6(-5)^2 + 9(-5) - 2 = -125 + 150 - 45 - 2 = -22$

$f(-3) = (-3)^3 + 6(-3)^2 + 9(-3) - 2 = -27 + 54 - 27 - 2 = -2$

$f(-1) = (-1)^3 + 6(-1)^2 + 9(-1) - 2 = -1 + 6 - 9 - 2 = -6$

$f(3) = (3)^3 + 6(3)^2 + 9(3) - 2 = 27 + 54 + 27 - 2 = 106$

x	-5	-3	-2	-1	0	3
$f'(x)$	+	0	-	0	+	+
$I\ or\ D$	I		D		I	I
Local Maximum Or Minimum		Local Maximum		Local Minimum		
$f(x)$	-22	-2		-6		106
Absolute Maximum or Minimum	Absolute Minimum					Absolute Maximum



Anti - Derivatives

If $f(x) = x^2$, then $f'(x) = 2x$.

The derivative of $f(x)$ is $f'(x)$.

If $g'(x) = 3x^2$, the $g(x) = x^3$.

The antiderivative of $g'(x)$ is $g(x)$.

Find the antiderivative of the following:

$f'(x) = -\frac{5}{x^6}$ $\quad f(x) = x^{-5} + C$

$f'(x) = \frac{4}{x}$ $\quad f(x) = 4lnx + C$

$f'(x) = \cos(2x)$ $\quad f(x) = \frac{1}{2}\sin(2x) + C$

$f'(x) = \frac{7}{1+x^2}$ $\quad f(x) = 7tan^{-1}x$