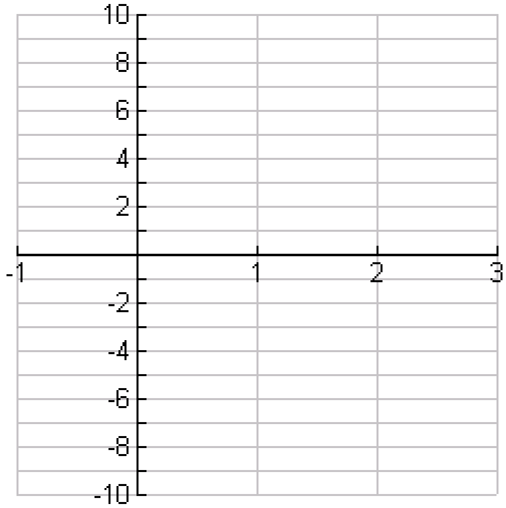
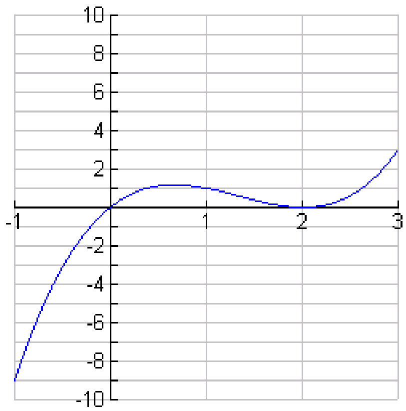
Calculus 4.1

Graph $y = x^3 - 4x^2 + 4x$ over the x boundary of [-1, 3].





What is the absolute lowest y value? -9 What x value corresponds to that y value? -1

The absolute minimum value occurs at $f(c)$ when $f(c) \leq f(x)$ for all x values in the domain.

Another name for absolute minimum is the global minimum.

What is the absolute highest y value? 3 What x value corresponds to that y value? 3

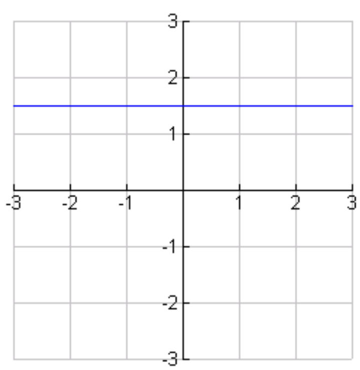
The absolute maximum value occurs at $f(c)$ when $f(c) \geq f(x)$ for all x values in the domain.

Another name for absolute maximum is the global maximum.

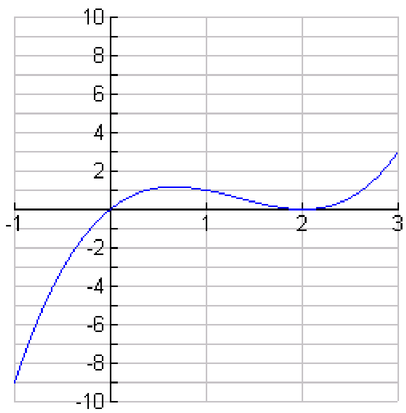
Absolute maximum and minimum values are also called absolute extrema.

The Extreme Value Theorem says that in a CC curve, $f(x)$ will have an absolute maximum and absolute minimum.

Try to give an example where this does not seem to be true but is.



This seems to not work but it does have a maximum and minimum value of 1.5 .



At what points are there horizontal tangents?

Horizontal tangents occur when the derivative equals 0.

Find the derivative of $y = x^3 - 4x^2 + 4x$.

$y' = 3x^2 - 8x + 4$

$y' = (3x - 2)(x - 2)$

$0 = (3x - 2)(x - 2)$

$x = \frac{2}{3}, 2$

Points where $f'(x) = 0$ or $f'(x)$ is undefined are called critical points.

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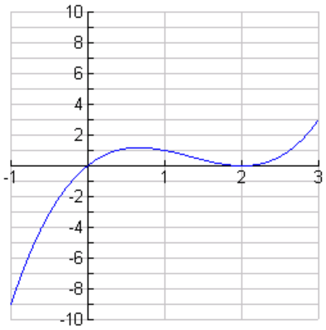
Points where the y value is greater than anything close to it are called local maximum values.

Points where the y value is less than anything close to it are called local minimum values.

Another word for local maximum or minimum is local extrema.



Where are the local or relative extrema of this graph?

The local extrema are at $x = \frac{2}{3}$ and $x = 2$.

There is a local maximum at $\left(\frac{2}{3}, \frac{32}{27}\right)$.

There is a local minimum at $(2, 0)$.

Theorem to remember If a function f has a local maximum or local minimum at a point c and if $f'(c)$ exists, then $f'(c) = 0$.

Technical notes:

When the problem asks for the maximum or minimum value, they want the y value.

If the problem asks for the maximum or minimum point, they want (x, y).

If the problem asks where does the maximum or minimum point occur, they want the x value.

Identify the critical points and determine the local extreme values for $f(x) = x\sqrt{4-x^2}$.

The critical points occur when $f' = 0$ or undefined.

$f'(x) = \sqrt{4-x^2} + x\left(\frac{-2x}{2\sqrt{4-x^2}}\right) = \frac{\sqrt{4-x^2}}{1} - \frac{x^2}{\sqrt{4-x^2}}$ Do diagonal-diagonal-denominator.

$f'(x) = \frac{4-x^2-x^2}{\sqrt{4-x^2}} = \frac{4-2x^2}{\sqrt{4-x^2}}$

Side bar notes with a fraction:

To find where it equals 0, let the numerator = 0.

To find where it is undefined, let the denominator = 0.

$f' = 0$ when $4 - 2x^2 = 0 \quad 4 = 2x^2 \quad x^2 = 2 \quad x = \pm\sqrt{2}$

f' is undefined when $\sqrt{4-x^2} = 0 \;\; 4 - x^2 = 0 \quad 4 = x^2 \quad x = \pm 2$

Let's graph it to get a feel for the look.

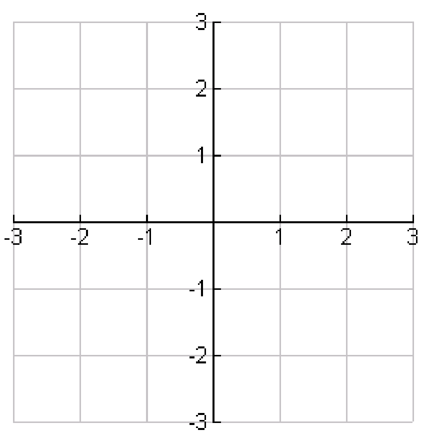
The zeros occur at $x = 0$ or $x = \pm 2$.

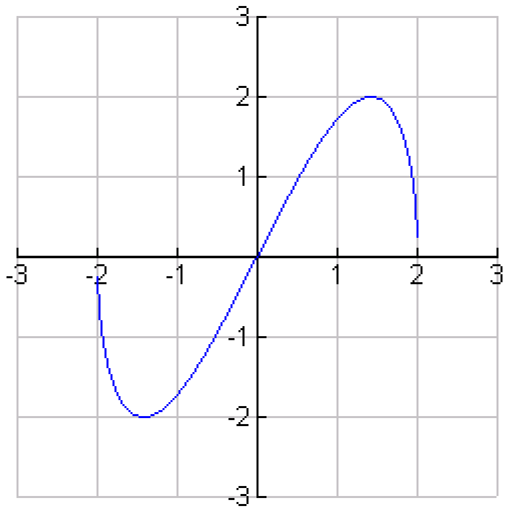
The domain of this problem is [-2, 2]

When $x = \sqrt{2},\ f(\sqrt{2}) = \sqrt{2}\sqrt{4-(\sqrt{x})^2} = \sqrt{2}\sqrt{2} = 2$

When $x = -\sqrt{2},\ f(-\sqrt{2}) = -\sqrt{2}\sqrt{4-(-\sqrt{x})^2} = -\sqrt{2}\sqrt{2} = -2$

Graph it here. Think of $\sqrt{2} = 1.4\ and - \sqrt{2} = -1.4$.





The critical points are $(\sqrt{2}, 2)$ & $(-\sqrt{2}, -2)$.

The absolute maximum value is 2 and the absolute minimum value is -2.

The local maximum value is 0 and the local minimum value is 0.