New Chapter 5.1 – 5.4 Review

The absolute minimum value occurs at $f(c)$ when $f(c) \leq f(x)$ for all x values in the domain.

Another name for absolute minimum is the global minimum.

The absolute maximum value occurs at $f(c)$ when $f(c) \geq f(x)$ for all x values in the domain.

Another name for absolute maximum is the global maximum.

Absolute maximum and minimum values are also called absolute extrema.

The Extreme Value Theorem says that in a CC curve, $f(x)$ will have an absolute maximum and absolute minimum.

Horizontal tangents occur when the derivative equals 0.

Points where $f'(x) = 0$ or $f'(x)$ is undefined are called critical points.

Points where the y value is greater than anything close to it are called local maximum values.

Points where the y value is less than anything close to it are called local minimum values.

Another word for local maximum or minimum is local extrema.

Theorem to remember If a function f has a local maximum or local minimum at a point c and if $f'(c)$ exists, then $f'(c) = 0$.

Side bar notes with a fraction:

To find where it equals 0, let the numerator = 0.

To find where it is undefined, let the denominator = 0.

The Mean Value Theorem says that if you take two points and draw the line between them, there has to be a point whose tangent line is parallel to the line between the two points.

$$f'(c) = \frac{y_2 - y_1}{x_2 - x_1}$$

The derivative at c equals the slope between (x_1, y_1) & (x_2, y_2).

Increasing and Decreasing Functions

If $f'(x) > 0$, the function is increasing.

If $f'(x) < 0$, the function is decreasing.

THE CHART

Put the x-value boundary endpoints or the x-value domain endpoints in the chart.

Find the derivative.

Find all points where the derivative equals 0 or is undefined.

Make up values to the left and right of the points where the derivative is 0.

Plug in the x values to $f'(x)$ to see if $f'(x) > 0$ or $f'(x) < 0$

Note if the function is increasing or decreasing

Determine local maximum or minimums.

Plug in x values for endpoints and local maximum or minimum to find the absolute max or min.

Anti - Derivatives

If $f(x) = x^2$, then $f'(x) = 2x$.

The derivative of $f(x)$ is $f'(x)$.

If $g'(x) = 3x^2$, then $g(x) = x^3$.

The antiderivative of $g'(x)$ is $g(x)$.

The First Derivative Test

At a critical point c, if f' changes from $+\ to\ -$, then f has a local maximum value at c.

At a critical point c, if f' changes from $-\ to\ +$, then f has a local minimum value at c.

Concavity

Concavity is determined by the second derivative of a function.

If $y'' > 0$, the function is concave up. Hooking up.

If $y'' < 0$, the function is concave down. Hooking Down

Point of Inflection

A point of inflection is where the concavity changes.

Special note:

If there is a point of inflection, the second derivative at that point will be 0 or undefined.

If the second derivative is 0 or undefined at a point, it does not have to be a point of inflection.

The Second Derivative Test

When the first derivative equals 0 and $y'' > 0$, the point is a local minimum.

When the first derivative equals 0 and $y'' < 0$, the point is a local minimum.

$f(x) = x^3 - 3x^2 - 24x + 3$ over the interval [-5, 10]

Find $f'(x)$ *and* $f''(x)$.

Create a chart that helps you find the absolute extrema and local extrema values. Include all critical points, increasing and decreasing intervals, points of inflection and concavity. Sketch the graph using your information.

$f'(x) = 3x^2 - 6x - 24$ $\qquad$ $f''(x) = 6x - 6$

x	-5	-2	0	1	4	10
$f'(x)$	+	0	-	-	0	+
I, D, LMA or LMI	I	L MAX	D	D	L MIN	I
$f(x)$	-77	31			-77	463
ABS MA ABS MI	ABS MIN				ABS MI	ABS MAX
$f''(x)$	-	-	-	0	+	+
U, D, POI	D	D	D	POI	U	U

$f(x) = x^3(4 - x) = 4x^3 - x^4$ over [-2, 4]

Find $f'(x)$ *and* $f''(x)$.

Create a chart that helps you find the absolute extrema and local extrema values. Include all critical points, increasing and decreasing intervals, points of inflection and concavity.

Sketch the graph using your information.

$f'(x) = 12x^2 - 4x^3$ $\qquad$ $f''(x) = 24x - 12x^2$

$0 = 12x^2 - 4x^3$ $\qquad$ $0 = 24x - 12x^2$

$0 = 4x^2(3 - x)$ $\qquad$ $0 = 12x(x - 2)$

$4x^2 = 0 \quad 3 - x = 0$ $\qquad$ $0 = 12x \quad x - 2 = 0$

$x = 0, 3$ $\qquad$ $x = 0, 2$

x	-2	0	1	2	3	4
$f'(x)$	+	0	+	+	0	-
I, D, LMAX or LMIN	I		I	I	L MAX	D
$f(x)$	-48	0		16	27	0
ABS MAX ABS MIN	ABS MIN				ABS MAX	L MIN
$f''(x)$	-	0	+	0	-	-
U, D, POI	D	POI	U	POI	D	D

The sum of two non-negative number is 16. Find the numbers if

a) the sum of their squares is as large as possible.
b) the sum of their squares is as small as possible.

Let $x =$ the first number and $16 - x =$ the second number.

$$f(x) = x^2 + (16 - x)^2$$

$$f'(x) = 2x - 2(16 - x) = 4x - 32$$

$$0 = 4x - 32 \quad x = 8$$

x	0	8	16
$f'(x)$	-	0	+
I, D, LMAX or LMIN	D	L MIN	I
$f(x)$	256	128	256
ABS MAX ABS MIN	ABS MAX	ABS MIN	ABS MAX

c) one number plus the square root of the other is as small as possible.

$$f(x) = \sqrt{x} + 16 - x$$

$$f'(x) = \frac{1}{2\sqrt{x}} - 1 = 0 \qquad \frac{1}{2\sqrt{x}} = 1 \quad 1 = 2\sqrt{x} \quad \frac{1}{2} = \sqrt{x} \quad \frac{1}{4} = x$$

x	0	$\frac{1}{9}$	$\frac{1}{4}$	16
$f'(x)$	Und	+	0	-
I, D, LMAX or LMIN			L MAX	D
$f(x)$	16		$16\frac{1}{4}$	4
ABS MAX ABS MIN	L MIN		ABS MAX	ABS MIN

When the numbers are $\frac{1}{4}$ & $15\frac{3}{4}$ the value is the largest.

When the numbers are 16 & 0, the value is the smallest.

The area of a rectangle is 25. Find the dimensions of the smallest perimeter that can be constructed.

Let $x = length$ and $\frac{25}{x} = width$.

The perimeter $f(x) = 2x + 2\left(\frac{25}{x}\right) = 2x + 50x^{-1}$

$f'(x) = 2 - 50x^{-2} = 0$ $\frac{50}{x^2} = 2$ $50 = 2x^2$ $x^2 = 25$ $x = \pm 5$ $\rightarrow x = 5$

x	0	1	5	100
$f'(x)$	Und	-	0	+
I, D, LMAX or LMIN	X	D	L MIN	I
$f(x)$	X	52	20	200.5
ABS MAX ABS MIN	X		ABS MIN	

When the sides are 5 & 5, the perimeter is the smallest.

Find the dimensions that would create the smallest area inscribed under the curve

$f(x) = 3cos2x$ with the x-axis at its base.

The area $A = 2x(3cos2x) = 6xcos2x$.

$A' = 6(cos2x - 2xsinx)$

$6(cos2x - 2xsinx) = 0\ at\ x = .43$

The dimensions are 2(.43) by 3cos(.86) = 1.68

Find the dimensions of the lightest square based, open-top, rectangular box that can be made with a volume of 4000 ft^3.

Let $x =$ the side of the square base.

Let $h =$ the height of the box.

$V = x^2h \quad 4000 = x^2h \quad \frac{4000}{x^2} = h$

Surface area = $4sides + the\ base = 4x\left(\frac{4000}{x^2}\right) + x^2 = SA$

$SA = \frac{16000}{x} + x^2 = 16000x^{-1} + x^2$

$SA' = -16000x^{-2} + 2x = 0$

$\frac{16,000}{x^2} = 2x \quad 16,000 = 2x^3 \quad 8,000 = x^3 \quad x = 20$

The dimensions are 20 by 20 by 10.