5.1 - 5.3 Review

The absolute minimum value occurs at $f(c)$ when $f(c) \leq f(x)$.

Another name for absolute minimum is a global minimum.

The absolute maximum value occurs at $f(c)$ when $f(c) \geq f(x)$.

Another name for absolute maximum is global maximum.

Absolute maximum and minimum values are also called global extrema.

The Extreme Value Theorem says that in a CC curve, there has to be a global maxmum and minimum.

Horizontal tangents occur when $f'(x) = 0$.

Critical points exist when $f'(x) = 0 \; or \; f'(x)$ is undefined.

Absolute maximums and minimums occur at critical points or endpoints.

Points where the y value is greater than anything close to it are local maximums.

Points where the y value is less than anything close to it are local minimums.

Another word for local maximum or minimum is relative maxi or min.

Theorem to remember. If a function f has a local maximum or local minimum at a point c and if $f'(c)$ exists, then $f'(c) = 0$.

If $f'(x) = \frac{ax+b}{cx-d}$ $f'(x) = 0\ at\ x = -\frac{b}{a}$ $f'(x)$ is undefined at $x = \frac{d}{c}$

What is the Mean Value Theorem in words?

The derivative at one point equals the slope between two points.

What is the Mean Value Theorem formula?

$f'(c) = \frac{y_1 - y_2}{x_1 - x_2}$

If $f'(x) > 0$, $f(x)$ is increasing.

If $f'(x) < 0$, $f(x)$ is decreasing.

If $f'(x)$ changes from positive to negative at $x = 3$, there is a local maximum at $x = 3$.

If $f'(x)$ changes from negative to positive at $x = 4$, there is a local minimum at $x = 4$.

If $f''(x) < 0$, $f(x)$ is concave down.

If $f''(x) > 0$, $f(x)$ is concave up.

If $f''(x)$ changes from negative to positive at $x = 7$, there is a point of inflection at $x = 7$.

If $f'(3) = 0, f''(3) = 9$, there is local minimum at $x = 3$.

If $f'(9) = 0, f''(9) = -17.345\pi$, there is a local maximum at $x = 9$.

A point of inflection is where the concavity changes.

Find the value of c that satisfies the Mean Value Theorem for the function $y = x^2 + 6x - 1$ over the interval [-2, 5].

Derivative equals slope. $x = -2, y = -9 \quad x = 5, y = 54$

$$2x + 6 = \frac{54 + 9}{5 + 2} = \frac{63}{7} = 9$$

$$2x = 3 \quad x = \frac{3}{2}$$

Find the anti-derivatives of

$f'(x) = -\frac{5}{x^3} = -5x^{-3} \qquad f(x) = \frac{5}{2}x^{-2} + C$

$f'(x) = \frac{4}{x} \qquad f(x) = 4lnx + C$

$f'(x) = \sin(7x) \qquad f(x) = -\frac{1}{7}\cos(7x) + C$

$f'(x) = \frac{-8}{1+x^2} \qquad f(x) = 8cot^{-1}x + C \quad or - 8tan^{-1}x + C$

$f(x) = x^3 - 3x^2 - 24x + 3$ over the interval [-5, 10]

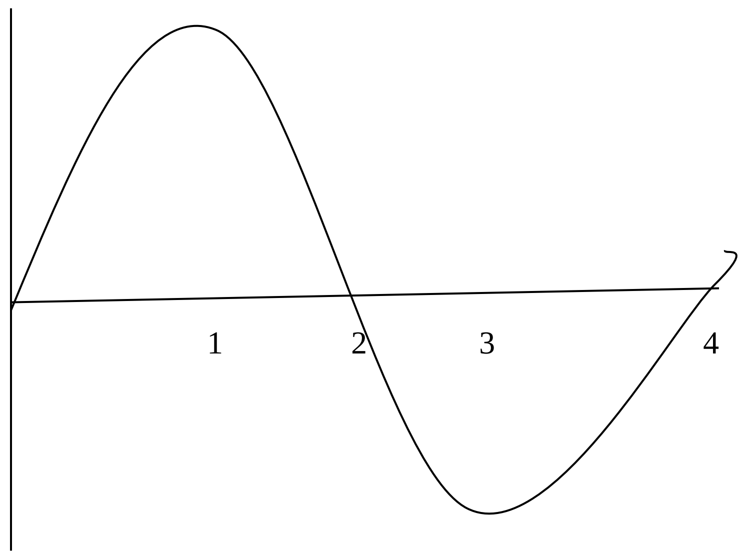
$f'(x) = 3x^2 - 6x - 24 = 0 \qquad 3(x^2 - 2x - 8) = 0 \quad x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0 \qquad x = 4, -2$

$f''(x) = 6x - 6 = 0 \quad 6x = 6 \quad x = 1$

x	-5	-2	1	4	10
$f'(x)$	+	0	-	0	+
I Or D	I		D		I
Local max or min		Max		Min	
$f(x)$	-77	31		-77	463
Absolute max or min	Min			Min	Max
$f''(x)$	-	-	0	+	+
U or D	U	U		D	D
POI			POI		

This is a $f(x)$ graph.



$f(x)$ is increasing from $(0, 2)$

$f(x)$ is decreasing from $(2, 4)$

The local max is at $x = 4$

The local min is at $x = 0$

The absolute max is at $x = 1$

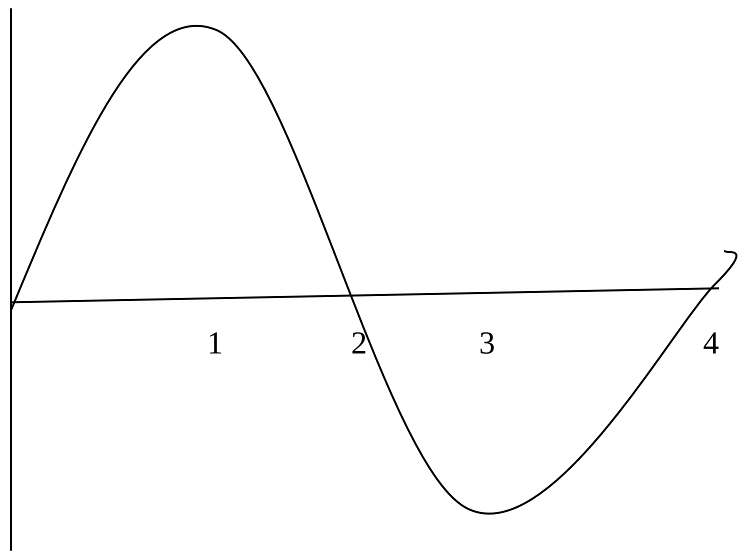
The absolute min is at $x = 3$

$f(x)$ is concave up from $(2, 4)$

$f(x)$ is concave down from $(0, 2)$

There is a point of inflection at $x = 2$

This is a $f'(x)$ graph.



$f(x)$ is increasing from $(0, 2)$

$f(x)$ is decreasing from $(2, 4)$.

The local max is at $x = 2$

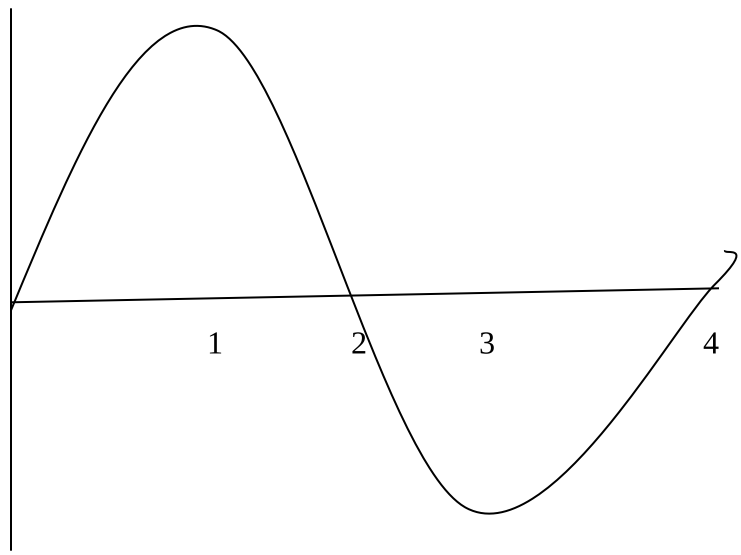
The local min is at $x = 0, 4$

$f(x)$ is concave up from $(0, 1) \cup (3, 4)$

$f(x)$ is concave down $(1, 3)$

There is a point of inflection at $x = 1, 3$

This is a $f''(x)$ graph.



$f(x)$ is concave up from $(0, 2)$.

$f(x)$ is concave down from $(2, 4)$.

There is a point of inflection at $x = 2$.

Concavity is determined by $f''(x)$.