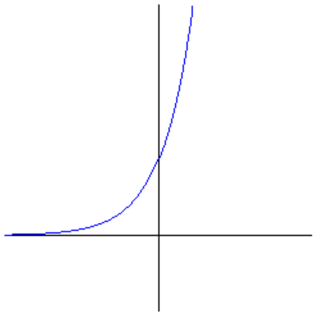
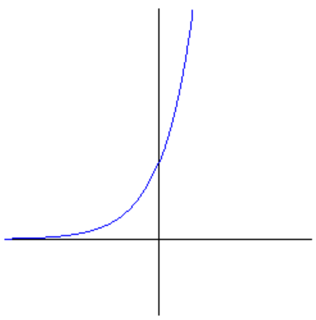
Calculus 4.4 Exponential and Logarithmic Functions

$y = e^x$

Graph it.



Pick points along the graph and graph the derivative.



What is the derivative of $y = e^x$?

Prove your answer using the definition of the derivative.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \frac{e^{h+x}-e^x}{h} = \frac{e^h e^x - e^x}{h} = \frac{e^x(e^h-1)}{h} = \mathrm{e}^{\mathrm{x}}\lim_{h\to 0}\frac{e^h-1}{h} = e^x$$

What is the derivative of $y = lnx$?

Start by changing this to exponential form.

$e^y = x.$

Take the derivative using implicit differentiation.

$y'e^y = 1$ $\qquad$ $y' = \frac{1}{e^y} = \frac{1}{x}$

What is the derivative of $y = \log_3 x$?

Start by changing this to exponential form.

$3^y = x$

Take the ln of both sides.

$ln3^y = lnx$

Use the properties of logarithms.

$yln3 = lnx$

Take the derivative of both sides.

$y'ln3 = \frac{1}{x}$

Divide by $ln3$.

$y' = \frac{1}{xln3}$

The general form of this type of function is if $y = \log_a x$ then $y' = \frac{1}{xlna}$.

What is the derivative of $y = 5^x$?

Take the ln of both sides.

$lny = ln5^x$

Use the properties of logarithms.

$lny = xln5$

Take the derivative of both sides.

You use implicit with the left side and remember that $ln5$ is just a number.

$\frac{y'}{y} = ln5$

Multiply both sides by y.

$y' = y\ln 5$

Go back to the original problem and see what y equals.

$y' = (ln5)5^x$

What is the general form to this type of problem?

If $y = a^x, y' = (lna)a^x$

Review

If $y = e^x, then\ y' = e^x$

If $y = lnx, then\ y' = \frac{1}{x}$

If $y = \log_a x,\ then\ y' = \frac{1}{(lna)x}$

If $y = a^x, then\ y' = (lna)a^x$

Chain Rule Form

If $y = e^{f(x)}, then\ y' = f'(x)e^x$

If $y = \ln f(x),\ then\ y' = \frac{f'(x)}{f(x)}$

If $y = \log_a f(x),\ then\ y' = \frac{f'(x)}{(lna)f(x)}$

If $y = a^{f(x)}, then\ y' = f'(x)a^{f(x)}$

Logarithmic Differentiation

If $8 = 3^x$, in order to solve for x, you would have to take the natural logarithm of both sides.

When you are taking a derivative of a function that has an x as a power, you need to use logarithmic differentiation. This means that you would take the natural logarithm of both sides and then take the derivative.

$y = x^x$

$lny = lnx^x$ Use the properties of logarithms to create:

$lny = xlnx$ Use implicit differentiation with the left side and a product rule on the right side.

$\frac{y\prime}{y} = x\left(\frac{1}{x}\right) + (1)lnx$ Simplify the right side.

$\frac{y\prime}{y} = 1 + lnx$ Multiply both sides by y which is x^x.

$y' = x^x(1 + lnx)$

Another example

$y = (\sin(5x))^{secx}$ Take the natural log of both sides.

$lny = \ln(\sin(5x))^{secx}$ Use the properties of logarithms.

$lny = secxln(sin5x)$ Implicit on the left side and product rule on the left. Chain rule everywhere.

$\frac{y\prime}{y} = secxtanxln(sin5x) + secx\left(\frac{cos5x}{sin5x}\right)(5)$ Trig identity.

$\frac{y\prime}{y} = secxtanxln(sin5x) + 5secxcot(5x)$ Multiply by y.

$y' = (\sin(5x))^{secx}(secxtanxln(sin5x) + 5secxcot(5x))$