Calculus 4.3

Find the derivative of $y = sin^{-1}x$.

$$y = sin^{-1}x \rightarrow x = siny$$

Take the derivative of $x = siny$.

$$1 = y'cosy$$

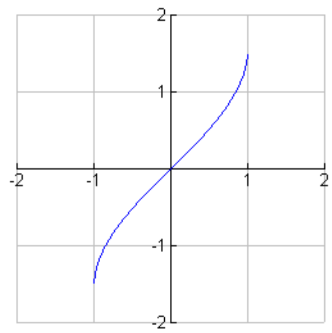
$$\frac{1}{cosy} = y'$$

$$sin^2y + cos^2y = 1$$

$$cos^2y = 1 - sin^2y$$

$$cosy = \pm\sqrt{1 - sin^2y}$$

This is the graph of $y = sin^{-1}x$.



The derivative of each x value is positive.

$$cosy = \sqrt{1 - sin^2y}$$

$$\frac{1}{\sqrt{1-sin^2y}} = y'$$

$$\frac{1}{\sqrt{1-x^2}} = y'$$

Find the derivative of $y = cos^{-1}x$.

$$y = cos^{-1}x \rightarrow x = cosy$$

Take the derivative of $x = cosy$.

$$1 = -y'siny$$

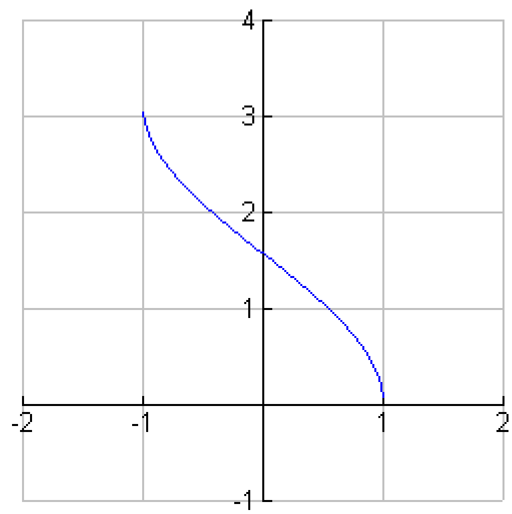
$$-\frac{1}{siny} = y'$$

$$sin^2y + cos^2y = 1$$

$$sin^2y = 1 - cos^2y$$

$$siny = \pm\sqrt{1 - cos^2y}$$

This is the graph of $y = cos^{-1}x$.



All of the derivatives of any x value are negative. My formula already has a negative sign in it. We must make the square root positive.

$$siny = \sqrt{1 - cos^2y}$$

$$-\frac{1}{\sqrt{1-cos^2y}} = y'$$

$$-\frac{1}{\sqrt{1-x^2}} = y'$$

Find the derivative of $y = tan^{-1}x$.

$$x = tany$$

Take the derivative of $x = tany$.

$$1 = y'sec^2y$$

$$\frac{1}{sec^2y} = y'$$

$$sin^2y + cos^2y = 1$$

$$tan^2y + 1 = sec^2y$$

$$\frac{1}{tan^2y+1} = y'$$

$$\frac{1}{x^2+1} = y'$$

Find the derivative of $y = cot^{-1}x$.

$$x = coty$$

Take the derivative of $x = coty$.

$$1 = -y'csc^2y$$

$$-\frac{1}{csc^2y} = y'$$

$$sin^2y + cos^2y = 1$$

$$1 + cot^2y = csc^2y$$

$$-\frac{1}{cot^2y+1} = y'$$

$$-\frac{1}{x^2+1} = y'$$

Find the derivative of $y = sec^{-1}x$.

$$y = sec^{-1}x \rightarrow x = secy$$

Take the derivative of $x = secy$.

$$1 = y' secytany$$

$$\frac{1}{secytany} = y'$$

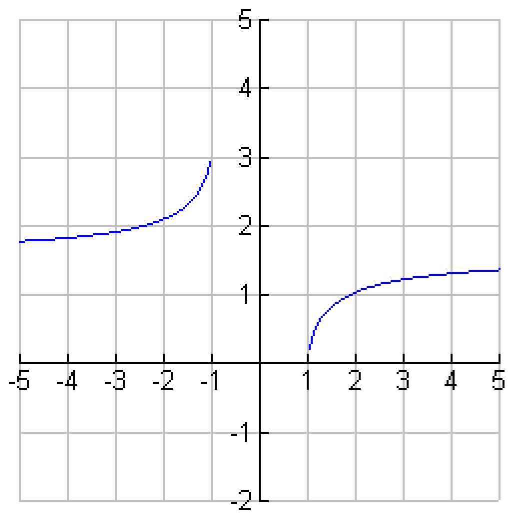
$$sin^2y + cos^2y = 1$$

$$tan^2y + 1 = sec^2y$$

$$tan^2y = sec^2y - 1$$

$$tany = \pm\sqrt{sec^2y - 1}$$

This is the graph of $y = sec^{-1}x$. The derivative at each value is positive.



$$tany = \sqrt{sec^2y - 1}$$

$$\frac{1}{secy\sqrt{sec^2y-1}} = y'$$

$$\frac{1}{|x|\sqrt{x^2-1}} = y'$$

Find the derivative of $y = csc^{-1}x$.

$$y = csc^{-1}x \rightarrow x = cscy$$

Take the derivative of $x = cscy$.

$$1 = -y' cscycoty$$

$$-\frac{1}{cscycoty} = y'$$

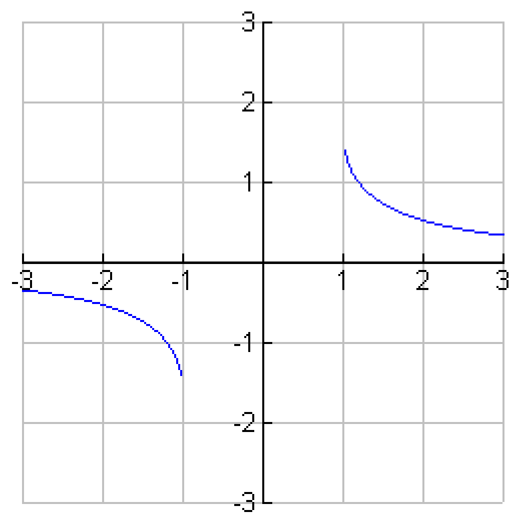
$$sin^2y + cos^2y = 1$$

$$1 + cot^2y = csc^2y$$

$$cot^2y = csc^2y - 1$$

$$coty = \pm\sqrt{csc^2y - 1}$$

This is the graph of $y = csc^{-1}x$. The derivative at each value is negative. Our answer has a negative sign in it. We just want the positive square root values.



$$coty = \sqrt{csc^2y - 1}$$

$$-\frac{1}{|x|\sqrt{x^2-1}} = y'$$

$$-\frac{1}{|x|\sqrt{x^2-1}} = y'$$

Review all of the inverse trigonometric derivatives.

$y = sin^{-1}x$ $\qquad$ $y' = \dfrac{1}{\sqrt{1-x^2}}$

$y = cos^{-1}x$ $\qquad$ $y' = -\frac{1}{\sqrt{1-x^2}}$

$y = tan^{-1}x$ $\qquad$ $y' = \frac{1}{x^2+1}$

$y = cot^{-1}x$ $\qquad$ $y' = \frac{1}{x^2+1}$

$y = sec^{-1}x$ $\qquad$ $y' = \frac{1}{|x|\sqrt{x^2-1}}$

$y = csc^{-1}x$ $\qquad$ $y' = -\frac{1}{|x|\sqrt{x^2-1}}$

Do it again in chain rule notation.

$y = sin^{-1} f(x)$ $$y' = \frac{f'(x)}{\sqrt{1 - (f(x))^2}}$$

$y = cos^{-1} f(x)$ $$y' = -\frac{f'(x)}{\sqrt{1-(f(x))^2}}$$

$y = tan^{-1} f(x)$ $$y' = \frac{f'(x)}{(f(x))^2+1}$$

$y = cot^{-1} f(x)$ $$y' = \frac{f'(x)}{(f(x))^2+1}$$

$y = sec^{-1} f(x)$ $$y' = \frac{f'(x)}{|f(x)|\sqrt{(f(x))^2-1}}$$

$y = csc^{-1} f(x)$ $$y' = -\frac{f'(x)}{|f(x)|\sqrt{(f(x))^2-1}}$$