Calculus 4.2

Take the derivative of $y = 6x^2 + 7x + 5$.

Using the shortcut, you get $y' = 12x + 7$.

The derivative of y is y'.

What if we have an expression with a y in it?

$xy + 3x - 7 = 0$ You have a product rule with the xy.

The derivative of x is 1and the derivative of y is y'.

$xy' + y + 3 = 0$ Solve for y'.

$xy' = -y - 3$

$y' = \frac{-y-3}{x} = -\frac{y+3}{x}$

What if the y is being raised to a power?

$y^2 \rightarrow 2yy'$

$y^3 \rightarrow 3y^2y'$

What if the y is involved in an operation?

$siny \rightarrow y'cosy$

$secy \rightarrow y'secytany$

Every time you have a y in the problem, we have to multiply by a y'.

Let’s get busy.

Find the derivative of $y^2 + \frac{1}{y} + 13y = 4x$

Rewrite it as $y^2 + y^{-1} + 13y = 4x$

$2yy' - y^{-2}y' + 13y' = 4$

$2yy' - \frac{y'}{y^2} + 13y' = 4$ — Multiply both sides of the equation by y^2.

$2y^3y' - y' + 13y^2y' = 4y^2$ — Factor out a y'.

$y'(2y^3 - 1 + 13y^2) = 4y^2$ — Solve for y'.

$y' = \frac{4y^2}{2y^3+13y^2-1}$

How do we know this works? Let’s take a derivative of an equation of a circle and check our derivative.

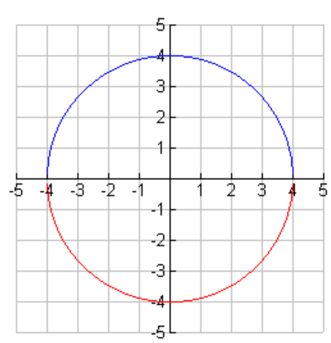
Find the derivative of $x^2 + y^2 = 16$

$2x + 2yy' = 0$

$2yy' = -2x$

$y' = -\frac{2x}{2y} = -\frac{x}{y}$

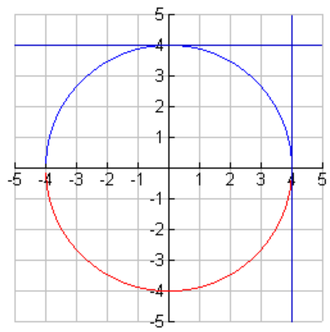
Let’s look at the graph and see if it makes sense.



What is the slope at (4, 0)?

What is the slope at (0, 4)?

If I draw tangent lines at those points, you see the slope at (0, 4) is 0 because the tangent line is horizontal. The slope at (4, 0) is undefined because the tangent line is vertical.



Let's find the second derivative of this equation.

$x^2 + y^2 = 16$

We already did the first derivative and got

$y' = -\frac{x}{y}$

Use the quotient rule to find y''.

$$y'' = -\frac{y - xy\prime}{y^2} = -\frac{y - x(-\frac{x}{y})}{y^2} = -\frac{y + \frac{x^2}{y}}{y^2} = -\frac{y^2 + x^2}{y^3} = -\frac{16}{y^3}$$ (We got the 16 from the original equation.)

More practice.

Solve for y'.

$siny + y^2 = 7x$

$y'cosy + 2yy' = 7$

$y'(cosy + 2y) = 7$

$y' = \frac{7}{cosy + 2y}$

Even more practice.

Solve for y'.

$$4cosxsiny = 1$$

$$-4sinxsiny + 4y'cosxcosy = 0$$

$$4y'cosxcosy = 4sinxsiny$$

$$y' = \frac{4sinxsiny}{4cosxcosy} = tanxtany$$

Find the slope of the tangent line at the point $(1, 1)$ for $(x^2 + y^2)^2 = 4xy$.

$$(x^2 + y^2)^2 = 4xy$$

$$2(x^2 + y^2)(2x + 2yy') = 4xy' + 4y$$

$$4x^3 + 4xy^2 + 4yy'x^2 + 4yy' = 4xy' + 4y$$

$$4yy'x^2 + 4yy' - 4xy' = 4y - 4x^3 - 4xy^2$$

$$4y'(yx^2 + y - x) = 4y - 4x^3 - 4xy^2$$

$$y' = \frac{4y-4x^3-4xy^2}{4(yx^2+y-x)} = \frac{y-x^3-xy^2}{yx^2+y-x} \ at\ (1,1) = \frac{1-1-1}{1+1-1} = -1$$

Let's get busy.

Find the derivative of $y^2 + \frac{1}{y} + 13y = 4x$.

$2yy' - \frac{y\prime}{y^2} + 13y' = 4$ Multiply both sides by y^2.

$$2y^3y' - y' + 13y^2y' = 4y^2$$

$$y'(2y^3 - 1 + 13y^2) = 4y^2$$

$$y' = \frac{4y^2}{2y^3 - 1 + 13y^2}$$