4.1 Chain Rule

There are two ways to take of a derivative of $y = (x+5)^2$. You can foil it out or use the power rule. They should both give the same answer.

$y = (x+5)^2$ $\qquad$ $y = (x+5)^2$

$y = x^2 + 10x + 25$ $\qquad$ $y' = 2(x+5) = 2x + 10$

$y' = 2x + 10$

Let's try another example. $y = (6x-2)^2$

$y = (6x-2)^2$ $\qquad$ $y = (6x-2)^2$

$y = 36x^2 - 24x + 4$ $\qquad$ $y' = 2(6x-2) = 12x - 4$

$y' = 72x - 24$

They are not the same answer. Which answer is right?

The left answer is correct.

What are we missing? If you multiply the right answer by 6, you will have the same answer.

Where did the 6 come from? That is the derivative of the function being squared (6x - 2).

Create a formula for this concept.

$$y = (f(x))^2 \qquad y' = 2\big(f(x)\big)f'(x)$$

Let's try another example.

$y = \sin(7x+9)$ There are two functions here, the sine function and a linear function.

The derivative of $\sin(anything)$ is $\cos(anything)$

The derivative of $7x + 9$ is 7.

$y' = 7(\cos(7x+9)$

Create a formula for this concept.

If $y = \sin(f(x))$, $y' = \cos(f(x))\, f'(x)$

Can we create a formula that will deal with any combination of functions?

If $y = f(g(x))$, $y' = f'(g(x))g'(x)$

This is called the chain rule.

Let's get fancy.

$y = tan^4(12x + 9)$

There are three functions here.

The function raised to the 4th power, the tangent function and the linear function.

The derivative of $(anything)^4$ is $4(anything)^3$ → $4(tan^3(12x + 9))$

The derivative of $\tan(anything)$ is $sec^2(anything)$ → $sec^2(12x + 9)$

The derivative of $12x + 9$ is 12

Multiply all of the parts together and you have your answer.

$$y' = 48tan^3(12x + 9)sec^2(12x + 9)$$

The TCC Problem

$y = (5x - 3)^6(2x^3 + 9)^8$

There are two function multiplied together so we will use the product rule.

$f(x) = (5x - 3)^6$ $\quad g(x) = (2x^3 + 9)^8$

$f'(x) = 6(5x - 3)^5(5)$ $\quad g'(x) = 8(2x^3 + 9)^7(6x^2)$

$f'(x) = 30(5x - 3)^5$ $\quad g'(x) = 48x^2(2x^3 + 9)^7$

$f(x)g'(x) + f'(x)g(x) = (5x - 3)^6(48x^2(2x^3 + 9)^7) + 30(5x - 3)^5(2x^3 + 9)^8$

The algebra from here is special.

Factor out the largest common factor from both terms.

$y' = (5x - 3)^5(2x^3 + 9)^7(48x^2(5x - 3) + 30(2x^3 + 9))$

$y' = (5x-3)^5(2x^3+9)^7(240x^3 - 144x^2 + 60x^3 + 270)$

Combine like terms and we are done.

$y' = (5x-3)^5(2x^3+9)^7(300x^3 - 144x^2 + 270)$

Quick Hitters

$y = (5x+7)^{\pi} \qquad y' = 5\pi(5x+7)^{\pi-1}$

$y = (7x^2+9)^8 \qquad y' = 8(7x^2+9)^7(14x) = 112x(7x^2+9)^7$

$y = (9x^3+8x+7)^4 \qquad y' = 4(9x^3+8x+7)^3(27x^2+8)$

$y = sin5x \quad y' = 5cosx$

$y = cos^3x \qquad y' = 3cos^2x(-sinx) = -3sinxcos^2x$

$y = 8tan^3(4x+1)$

$y' = 8\big(3tan^2(4x+1)sec^2(4x+1)(4)\big) = 96tan^2(4x+1)sec^2(4x+1)$

$y = .2sec^4(5x^2+7x-1)$

$y' = .2(4sec^3(5x^2+7x-1)(sec(5x^2+7x-1)tan(5x^2+7x-1))(10x+7)$

$y' = .2(10x+7)sec^4(5x^2+7x-1)tan(5x^2+7x-1)$