4.1 – 4.4 Review

Find the derivative of the following:

$y = b(f(x))^a$	$y' = abf'(x)(f(x))^{a-1}$
$y = f(x) + g(x)$	$y' = f'(x) + g'(x)$
$y = f(x)g(x)$	$y' = f'(x)g(x) + f(x)g'(x)$
$y = \frac{f(x)}{g(x)}$	$y' = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$
$y = e^{\sqrt{\pi}}$	$y' = 0$
$y = sinf(x)$	$y' = f'(x)cosf(x)$
$y = cosf(x)$	$y' = -f'(x)sinf(x)$
$y = tanf(x)$	$y' = f'(x)sec^2f(x)$
$y = cotf(x)$	$y' = -f'(x)csc^2f(x)$
$y = secf(x)$	$y' = f'(x)secf(x)tanf(x)$
$y = cscf(x)$	$y' = -f'(x)cscf(x)cotf(x)$
$y = f(g(h(x)))$	$y = f'\left(g\left(h(x)\right)\right)g'\left(h(x)\right)h'(x)$
$y = sin^{-1}f(x)$	$y' = \frac{f'(x)}{\sqrt{1-(f(x))^2}}$
$y = cos^{-1}f(x)$	$y' = -\frac{f'(x)}{\sqrt{1-(f(x))^2}}$
$y = tan^{-1}f(x)$	$y' = \frac{f'(x)}{1+(f(x))^2}$
$y = cot^{-1}f(x)$	$y' = -\frac{f'(x)}{1+(f(x))^2}$
$y = sec^{-1}f(x)$	$y' = \frac{f'(x)}{\lvert f(x)\rvert\sqrt{(f(x))^2-1}}$
$y = csc^{-1}f(x)$	$y' = \frac{-f'(x)}{\lvert f(x)\rvert\sqrt{(f(x))^2-1}}$
$y = e^{f(x)}$	$y' = f'(x)e^{f(x)}$

$y = a^{f(x)}$ $\quad$ $y = f'(x)a^{f(x)}lna$

$y = lnf(x)$ $\quad$ $y = \frac{f'(x)}{f(x)}$

$y = \log_a f(x)$ $\quad$ $y = \frac{f'(x)}{(lna)f(x)}$

Find the derivatives.

1) $y = sin(4x)$

$y' = 4\cos(4x)$

2) $y = sin^3(5x)$

$y' = 3\,sin^2(5x)\cos(5x)(5)$

$y' = 15sin^2(5x)\cos(5x)$

3) $y = sin^3(6xtanx)$

$y' = 3sin^2(6xtanx)\cos(6xtanx)(6tanx + 6xsec^x$

$y' = 18sin^2(6xtanx)\cos(6xtanx)(tanx + xsec^2x)$

4) $y = \left(\cos(\frac{3x}{secx})\right)^5$

$$y' = -5\left(\cos(\frac{3x}{secx})\right)^4 sin\left(\frac{3x}{secx}\right)\left(\frac{3secx - 3xsecxtanx}{(secx)^2}\right)$$

$$y' = -15\left(\cos(\frac{3x}{secx})\right)^4 sin\left(\frac{3x}{secx}\right)\left(\frac{1 - xtanx}{secx}\right)$$

Find and prove the derivative of

$y = sin^{-1}x \quad \rightarrow \quad x = siny \qquad 1 = y'cosy$

$$\frac{1}{cosy} = y'$$

$$sin^2y + cos^2y = 1$$

$$cos^2y = 1 - sin^2y$$

$$cosy = \pm\sqrt{1 - sin^2y}$$

$$cosy = \sqrt{1 - sin^2y}$$

$$\frac{1}{\sqrt{1-sin^2y}} = y' \qquad \frac{1}{\sqrt{1-x^2}} = y'$$

$y = cos^{-1}x \quad \rightarrow \quad x = cosy \qquad 1 = -y'siny$

$$-\frac{1}{siny} = y'$$

$$sin^2y + cos^2y = 1$$

$$sin^2y = 1 - cos^2y$$

$$siny = \pm\sqrt{1 - cos^2y}$$

$$siny = \sqrt{1 - cos^2y}$$

$$-\frac{1}{\sqrt{1-cos^2y}} = y'$$

$$-\frac{1}{\sqrt{1-x^2}} = y'$$

$y = tan^{-1}x \qquad x = tany \qquad\qquad 1 = y'sec^2y$

$\frac{1}{sec^2y} = y'$

$sin^2y + cos^2y = 1$

$tan^2y + 1 = sec^2y$

$\frac{1}{tan^2y+1} = y'$

$\frac{1}{x^2+1} = y'$

$y = cot^{-1}x \qquad x = coty \qquad\qquad 1 = -y'csc^2y$

$-\frac{1}{csc^2y} = y'$

$sin^2y + cos^2y = 1$

$1 + cot^2y = csc^2y$

$-\frac{1}{cot^2y+1} = y'$

$-\frac{1}{x^2+1} = y'$

Find the derivative of $y = sec^{-1}x$ $x = secy$

$1 = y'secytany$

$\frac{1}{secytany} = y'$

$sin^2y + cos^2y = 1$

$tan^2y + 1 = sec^2y$

$tan^2y = sec^2y - 1$

$tany = \pm\sqrt{sec^2y - 1}$

$tany = \sqrt{sec^2y - 1}$

$\frac{1}{secy\sqrt{sec^2y-1}} = y'$

$\frac{1}{|x|\sqrt{x^2-1}} = y'$

Find the derivative of $y = csc^{-1}x$ $x = cscy$ $1 = -y'cscycoty$

$-\frac{1}{cscycoty} = y'$

$sin^2y + cos^2y = 1$

$1 + cot^2y = csc^2y$

$cot^2y = csc^2y - 1$

$coty = \pm\sqrt{csc^2y - 1}$

$coty = \sqrt{csc^2y - 1}$

$-\frac{1}{|x|\sqrt{x^2-1}} = y'$

$-\frac{1}{|x|\sqrt{x^2-1}} = y'$

Find the derivative of $y = e^x$.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \frac{e^{h+x} - e^x}{h} = \frac{e^h e^x - e^x}{h} = \frac{e^x(e^h - 1)}{h} = \mathrm{e^x} \lim_{h \to 0} \frac{e^h - 1}{h} = e^x$$

Find and prove the derivative of $y = lnx$.

$e^y = x.$

$y^{'} e^y = 1 \qquad y^{'} = \frac{1}{e^y} = \frac{1}{x}$

Find and prove the derivative of $y = \log_a x$.

$a^y = x$

$lna^y = lnx$

$ylna = lnx$

$y' lna = \frac{1}{x}$

$y' = \frac{1}{xlna}$

Find and prove the derivative of $y = a^x$?

$lny = lna^x$

$lny = xlna$

$\frac{y'}{y} = lna$

$y' = y\mathrm{lna}$

$y' = (lna)a^x$

Great Problems

$y = sinx + (sinx)^{-1} + sin^{-1}x + sin(\mathrm{x}^{-1})$

$y = sinx + cscx + sin^{-1}x + \sin(\mathrm{x}^{-1})$

$y' = cosx - cscxcotx + \frac{1}{\sqrt{1-x^2}} - x^{-2}\cos(\mathrm{x}^{-1})$

$y' = cosx - cscxcotx + \frac{1}{\sqrt{1-x^2}} - \frac{\cos(\mathrm{x}^{-1})}{\mathrm{x}^2}$

$y = (tanx)^{sin^{-1}x}$

$lny = ln(tanx)^{sin^{-1}x}$

$lny = sin^{-1}xlntanx$

$sin^{-1}x$	$lntanx$
$\frac{1}{\sqrt{1-x^2}}$	$\frac{sec^2x}{tanx}$
	$\frac{\frac{1}{cos^2x}}{\frac{sinx}{cosx}}$
	$\frac{cosx}{sinxcos^2x}$
	$\frac{1}{sinxcosx}$
	$secxcscx$

$\frac{y'}{y} = sin^{-1}xsecxcscx + \frac{lntanx}{\sqrt{1-x^2}}$

$y' = y(sin^{-1}xsecxcscx + \frac{lntanx}{\sqrt{1-x^2}})$

$y' = (tanx)^{sin^{-1}x}(sin^{-1}xsecxcscx + \frac{lntanx}{\sqrt{1-x^2}})$

$y = \sqrt{1-x^2} + xsin^{-1}x$

$y' = \frac{1}{2}(1-x^2)^{-\frac{1}{2}}(-2x) + \frac{x}{\sqrt{1-x^2}} + sin^{-1}x$

$y' = -\frac{x}{\sqrt{1-x^2}} + \frac{x}{\sqrt{1-x^2}} + sin^{-1}x = sin^{-1}x$

$$y^2 + xy + \frac{1}{y^2} = 8x$$

$$2yy' + xy' + y - \frac{2y'}{y^3} = 8$$

Multiply byy^3.

$$2y^4y' + xy^3y' + y^4 - 2y' = 8y^3$$

$$2y^4y' + xy^3y' - 2y' = 8y^3 - y^4$$

$$y'(2y^4 + xy^3 - 2) = 8y^3 - y^4$$

$$y' = \frac{8y^3 - y^4}{2y^4 + xy^3 - 2}$$

Find $y"$ if $x^2 - y^2 = 16$.

$$2x - 2yy' = 0$$

$$-2yy' = -2x$$

$$y' = \frac{-2x}{-2y} = \frac{x}{y}$$

$$y" = \frac{y - xy\prime}{y^2}$$

$y" = \frac{y - x(\frac{x}{y})}{y^2}$ multiply by $\frac{y}{y}$

$$y" = \frac{y^2 - x^2}{y^3} = -\frac{x^2 - y^2}{y^3} = -\frac{16}{y^3}$$

Quick Review Problems

$y = \ln(sinx)$ $y' = \frac{cosx}{sinx} = cotx$

$y = e^{tanx}$ $y' = (sec^2x)(e^{tanx})$

$y = \log_3(tan^{-1}x) \quad y' = \frac{1}{ln3(1+x^2)}$

$y = sin^{-1}(x^3) \quad y' = \frac{3x^2}{\sqrt{1-(x^3)^2}} = \frac{3x^2}{\sqrt{1-x^6}}$

$y = sec^{-1}(8x-1) \quad y' = \frac{8}{|8x-1|\sqrt{(8x-1)^2-1}}$

$y = cot^{-1}(e^x) \quad y' = -\frac{e^x}{1+(e^x)^2} = -\frac{e^x}{1+e^{2x}}$

$y = 5^{xsiny} \quad y' = (y'cosy + xsiny)5^{xsiny}$

$y' - y'cosy5^{xsiny} = (xsiny)5^{xsiny}$

$y'(1 - cosy5^{xsiny}) = (xsiny)5^{xsiny}$

$y' = \frac{(xsiny)5^{xsiny}}{1-cosy5^{xsiny}}$