10.1 Power Series

An infinite series is defined as an expression in the form of

$$a_1 + a_2 + a_3 + \cdots + a_k + .. = \sum_{k=1}^{\infty} a_k$$

If the sum has a limit as $n \to \infty$, we say that it converges to the sum S.

If it does not, we say that the series diverges.

Use your calculator to see if these approach a value.

2nd stat, math, sum

2nd stat, ops, seq,

$sum(seq((f(x), x, 1, 100)$

$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \cdots$ approaches 1

$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$ approaches 4.197

Theorems to consider:

If an infinite series converges, then $\lim_{k\to\infty} a_k = 0$

If $\lim_{k\to\infty} a_k = 0$, then the series (you do not know).

If $\lim_{k\to\infty} a_k \neq 0$, then the series diverges.

What kind of series is $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots$? Geometric

What does it converge to? 1

$\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\frac{1}{81}+\cdots$ converges to $\frac{1}{2}$

$\frac{3}{4}+\frac{9}{16}+\frac{27}{64}+\frac{81}{256}+\cdots$ converges to 3

$\frac{5}{2}+\frac{25}{4}+\frac{125}{8}+\frac{625}{16}+\cdots$ converges to infinity

A geometric series will converge to $\frac{a_1}{1-r}$ if $-1<r<1$

$$1+x+x^2+x^3+x^4+\cdots=\frac{1}{1-x}$$

Put both of these in the calculator and look at the graphs. When do the graphs match? When $-1<x<1$.

$$1-x+x^2-x^3+x^4+\cdots=\frac{1}{1+x}$$

Put both of these in the calculator and look at the graphs. When do the graphs match? When $-1<x<1$.

$$1+x^2+x^4+x^6+x^8+\cdots=\frac{1}{1-x^2}$$

Put both of these in the calculator and look at the graphs. When do the graphs match? When $-1<x<1$.

$$x+x^2+x^3+x^4+\cdots=\frac{x}{1-x}$$

Put both of these in the calculator and look at the graphs. When do the graphs match?

$$1+2x+4x^2+8x^3+16x^4+\cdots=\frac{1}{1-2x}$$

Put both of these in the calculator and look at the graphs. When do the graphs match? When $-1<x<1$.

If $y = 1 + x + x^2 + x^3 + x^4 + \cdots = \frac{1}{1-x}$

then $y' = \frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3$ $=$

Put both of these in the calculator and look at the graphs. When do the graphs match? When $-1 < x < 1$.

If $y = 1 - x + x^2 - x^3 + x^4 + \cdots = \frac{1}{1+x}$

then $\int y = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5} = \int \frac{1}{\mathrm{x}} \mathrm{dx} = \ln(1 + x)$

Put both of these in the calculator and look at the graphs. When do the graphs match? When $-1 < x < 1$.

$$f(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \frac{x^n}{n!} + \cdots$$

$$f'(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \frac{x^n}{n!} + \cdots$$

$$\int f(x) = x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots \frac{x^n}{n!} + \cdots + C$$

$f(0) = 1$

What is $f(x)$? e^x

Put both of these in the calculator and look at the graphs. When do the graphs match?