More Review for Chapter 9.1 – 9.4

1) $lim_{x\to 0}\frac{1-e^x}{x} = lim_{x\to 0}\frac{-e^x}{1} = -e^0 = -1$

2) $lim_{x\to 0}\frac{cos2x - cosx}{sin^2x} = lim_{x\to 0}\frac{-2sin2x + sinx}{2sinxcosx} = lim_{x\to 0}\frac{-4cosx + cosx}{2sinx(-sinx) + 2cosxcosx}$

$lim_{x\to 0}\frac{-4cosx + cosx}{2sinx(-sinx) + 2cosxcosx} = \frac{-3}{2}$

3) $lim_{x\to 0}x^2lnx = lim_{x\to 0}\frac{lnx}{\frac{1}{x^2}} = lim_{x\to 0}\frac{\frac{1}{x}}{-\frac{2}{x^3}} = lim_{x\to 0}\frac{1}{x}\cdot\frac{x^3}{-2} =$

$lim_{x\to 0} - \frac{x^2}{2} = 0$

4) $lim_{x\to 0^+}(sinx)^{tanx}$ $\quad y = sinx^{tanx}$ $\quad lny = tanxln(sinx)$

$lny = lim_{x\to 0^+}\frac{\ln(sinx)}{cotx} = lim_{x\to 0}\frac{\frac{cosx}{sinx}}{-csc^2x} = lim_{x\to 0}\frac{cosx}{sinx}\cdot\frac{-sin^2x}{1} =$

$lim_{x\to 0} - cosxsinx = 0 \quad lny = 0 \quad y = 1$

5) $lim_{x\to\infty}(1 + \frac{1}{x})^x$ $\quad y = (1 + \frac{1}{x})^x$ $\quad lny = lim_{x\to\infty}xln\left(1 + \frac{1}{x}\right) =$

$lim_{x\to\infty}\frac{\ln\left(1 + \frac{1}{x}\right)}{\frac{1}{x}} = lim_{x\to\infty}\frac{\frac{-\frac{1}{x^2}}{1 + \frac{1}{x}}}{-\frac{1}{x^2}} = lim_{x\to\infty}\frac{-\frac{1}{x^2}}{1 + \frac{1}{x}}\cdot\frac{-x^2}{1} = lim_{x\to\infty}\frac{1}{1 + \frac{1}{x}} = 1$

$lny = 1 \quad y = e$

6) $lim_{x\to 0}\left(\frac{1}{x} - \frac{1}{sinx}\right) = lim_{x\to 0}\frac{sinx - x}{xsinx} = lim_{x\to 0}\frac{cosx - 1}{sinx + xcosx}$

$lim_{x\to 0}\frac{-sinx}{cosx + cosx - xsinx} = 0$

7) $\int_1^5 \frac{1}{x-3} dx = lim_{b\to 3^-} \int_1^b \frac{1}{x-3} dx + lim_{b\to 3^+} \int_b^5 \frac{1}{x-3} dx =$

$$lim_{b\to 3^-} ln|x-3| \Big|_1^b + lim_{b\to 3^+} ln|x-3| \Big|_b^5 =$$

$$lim_{b\to 3^-} ln|b-3| - ln|1-3| + lim_{b\to 3^+} ln|b-3| - ln|5-3|$$

This goes to ∞, so the integral diverges.

8) $\int_0^1 \frac{1}{x} dx = lim_{b\to 0} \int_b^1 \frac{1}{x} dx = lim_{b\to 0} lnx\Big|_b^1 = lim_{b\to 0} ln1 - lnb$

This goes to ∞, so the integral diverges.

9) $\int_0^\infty e^{-x} cosxdx$ GREID

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$cosx$	$+$	e^{-x}
$-sinx$	$-$	$-e^{-x}$
$-cosx$		e^{-x}

$$lim_{b\to\infty} \int_0^b e^{-x} cosxdx = -cosx(e^{-x}) + sinx(e^{-x}) - \int_0^\infty e^{-x} cosxdx$$

$$lim_{b\to\infty} 2\int_0^b e^{-x} cosxdx = lim_{b\to\infty} - cosx(e^{-x}) + sinx(e^{-x})$$

$$lim_{b\to\infty} \int_0^b e^{-x} cosxdx = lim_{b\to\infty} \frac{(-cosx+sinx)}{2e^b} = 0$$

10) Let R be the region in the first quadrant under $xy = 9$ and to the right of $x = 1$. Find the volume generated by revolving R about the x-axis.

$y = \frac{9}{x}$ pancakes $A = \pi r^2$ $r = \frac{9}{x}$

$\pi \int_1^b (\frac{9}{x})^2 dx = lim_{b\to\infty} \pi \int_1^b \frac{81}{x^2} dx = lim_{b\to\infty} \pi \int_1^b 81x^{-2} dx =$

$lim_{b\to\infty} \frac{-81\pi}{x} \Big|_1^b = -\frac{81\pi}{b} + \frac{81\pi}{1} = 81\pi$

11) $\int_{\frac{1}{\pi}}^{\infty} \frac{1}{x^2} sin \frac{1}{x} dx$ u substitution

$u = \frac{1}{x}$ $du = -\frac{1}{x^2} dx$ $- du = \frac{1}{x^2} dx$

$-\int sinudu = cosu = lim_{b\to\infty} cos \frac{1}{x} \Big|_{\frac{1}{\pi}}^{b} = lim_{b\to\infty} cos \frac{1}{b} - cos \frac{1}{\frac{1}{\pi}}$

$lim_{b\to\infty} cos \frac{1}{b} - lim_{b\to\infty} cos\pi = 1 - (-1) = 2$

Write a formula for the nth term a_n of the sequence and determine its limit(if it exists).

12) $1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7} \ldots$ $a_n = \frac{1}{2n-1}$ $\to$ 0

13) $\frac{1}{4}, \frac{2}{5}, \frac{3}{6}, \frac{4}{7} \ldots$ $a_n = \frac{n}{n+3}$ $\to$ 1

14) $1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \ldots$ $a_n = (-1)^{n+1} \frac{1}{n} \to 0$

15) $-\frac{1}{4}, \frac{1}{9}, -\frac{1}{16}, \frac{1}{25}$ $\quad a_n = (-1)^n \frac{1}{(n+1)^2} \quad \rightarrow \quad 0$

16) $1, \frac{2}{2}, \frac{3}{6}, \frac{4}{24}, \frac{5}{120}, \frac{6}{720}$ $\quad a_n = \frac{n}{n!} \quad \rightarrow 0$

Determine whether the sequence converges, and if it does, find the limit.

17) $a_n = \cos(n\pi)$ diverges as it alternates between -1 & 1

18) $a_n = \frac{n}{e^n}$ $\qquad lim_{n\to\infty} \frac{n}{e^n} = 0$

19) $a_n = (1 + \frac{1}{n})^n$ $\quad y = (1 + \frac{1}{n})^n \quad lny = nln\left(1 + \frac{1}{y}\right)$

$$lim_{n\to\infty} \frac{ln\left(1 + \frac{1}{n}\right)}{\frac{1}{n}} = lim_{n\to\infty} \frac{\frac{-\frac{1}{n^2}}{1 + \frac{1}{n}}}{-\frac{1}{n^2}} = \frac{-\frac{1}{n^2}}{1 + \frac{1}{n}} \cdot -n^2 = lim_{n\to\infty} \frac{1}{1 + \frac{1}{n}} = 1$$

$lny = 1 \quad y = e$

20) $a_n = 2n\, sin\frac{\pi}{n}$ $\qquad lim_{n\to\infty} 2nsin\frac{\pi}{n} = lim_{n\to\infty} \frac{sin\frac{\pi}{n}}{\frac{1}{2n}} = \frac{-\frac{\pi}{n^2} cos\frac{\pi}{n}}{-\frac{1}{2n^2}}$

$$lim_{n\to\infty} 2\pi cos\frac{\pi}{n} = 2\pi$$

21) $a_n = \frac{2n^5 - 3n}{n^2 + 6n - 2}$ Bill Gates ∞ Diverges.

22) $a_n = \frac{\sqrt[3]{2n^5 - n^2 + 7}}{n^2 + 1}$ Bill Gates $\frac{n^{\frac{5}{3}}}{n^2} = \frac{1}{n^{\frac{1}{3}}} = 0$