Error

Actual - approximation

Geometric series

Find the error using just the first three terms of

First three terms =

Actual =

Error = .423280

Alternating series

Find the error of using just the first three terms of

The error is less than the next term =

Find the error of using just the first four terms of

The error is less than the next term = .

Maclaurin’s Theorem

Maclaurin Series with Lagrange Remainder

Lagrange Error Bound

If is the Lagrange remainder after partial sum , and is an upper bound for on the interval between , then

A Lagrange error bound for is given by

is the largest possible value for the ) st derivative between

There can be many answers for the error bound.

Example 1 when I give you the

Suppose that the first five terms of the Maclaurin series for for a function are

1. Find the value of the fourth derivative, .

2.352

1. Given that , show that the remainder of the series after the first five terms has an absolute value less than

Example \* when I give you the

Suppose that the first six terms of the Maclaurin series for for a function are

1. Find the value of the fifth derivative, .
2. Given that , show that the remainder of the series after the first five terms has an absolute value less than

Example 1 when you have to figure out the M

1. Find the first four non-zero terms of the Maclaurin series for the function

.

1. Find using this approximation using 6 places past the decimal.

1. What is the actual value of ?
2. What is the actual error using problem b as your approximation.
3. Find the error bound of problem b using the fact that this is an alternating series.
4. Find the Lagrange error bound of this approximation.

Example 2 when you have to figure out the M

1. Find the first four non-zero terms of the Maclaurin series for the function

1. Find using this approximation using 6 places past the decimal.
2. What is the actual value of ?
3. What is the actual error using problem b as your approximation.
4. Find the error bound of problem b using the fact that this is an alternating series.
5. Find the Lagrange error bound of this approximation.

More Examples

1. Find the fourth degree Taylor polynomial for about
   1. Use your polynomial to approximate the value of Use 6 places.

* 1. Use the Lagrange Error bound to determine the accuracy of the approximation.

1. Write the second-degree polynomial for centered at

2.4) Use your polynomial to approximate Use 6 places.

2.8) Find a Lagrange error bound for the maximum error on the approximation.

where and you want to create the largest possible value

creates the largest possible value

Let be a function that has derivatives of all orders for all real numbers Assume that and

for all in the closed interval [5, 5.2].

3) Find the third-degree Taylor polynomial about for

3.6) Use this approximation to estimate

3.97) What is the maximum possible error in making this estimate?

4) Let be a function that has derivatives of all orders. Assume that

and the graph of

on [3, 3.7] is shown below.

(3.4, 6)

(3, 4)

(3.7, 2)

1. Find the Taylor polynomial about for the function
2. Use your answer to approximate the value of .
3. What is the maximum possible error for this approximation ?

5) Let be the function defined by .

1. Find the second-degree Taylor polynomial about for the function .

1. Use your answer to approximate the value of .

1. Find a bound on the error for the approximation.

creates the largest M

1. Find the value of

6) Let be the function given by and let be the fourth-

degree Taylor polynomial for about

1. Find .
2. Use the Lagrange error bound to show that .