

Practice for U substitution Integrals

$$\int u du = \frac{u^2}{2} + C$$

$$\int u^3 du = \frac{u^4}{4} + C$$

$$\int u^n du = \frac{u^{n+1}}{n+1} + C$$

$$\int \frac{du}{u^2} = -\frac{1}{u} + C$$

$$\int \frac{du}{u^5} = -\frac{1}{4u^4} + C$$

$$\int \frac{du}{u^n} = -\frac{1}{(n-1)u^{n-1}}$$

$$\int \sqrt{u} du = \frac{2}{3} u^{\frac{3}{2}} + C$$

$$\int \sqrt[3]{u} du = \left(\frac{3}{4}\right) u^{\frac{4}{3}} + C$$

$$\int \sqrt[n]{u} du = \frac{n}{n+1} u^{\frac{n+1}{n}} + C$$

$$\int \frac{du}{\sqrt{u}} = 2u^{\frac{1}{2}} + C$$

$$\int \frac{du}{\sqrt[3]{u}} = \frac{3}{2} u^{\frac{2}{3}} + C$$

$$\int \frac{du}{\sqrt[n]{u}} = \left(\frac{n+1}{n}\right) u^{\frac{n+1}{n}} + C$$

$$\int \frac{du}{u} = \ln|u| + C$$

$$\int \frac{du}{u+7} = \ln|u+7| + C$$

$$\int \frac{du}{u-7} = \ln|u-7| + C$$

$$\int e^u du = e^u + C$$

$$\int \frac{du}{e^u} = -e^{-u} + C$$

$$\int \sin u du = -\cos u + C$$

$$\int \cos u du = \sin u + C$$

$$\int \sec^2 u du = \tan u + C$$

$$\int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} x + C$$

$$\int \frac{du}{1+u^2} = \tan^{-1} x + C$$

$$\int \frac{-3du}{1+u^2} = -3 \tan^{-1} u + C$$

Tricks in Integration

$$\int \frac{3+x}{x} dx = \int \frac{3}{x} dx + \int \frac{x}{x} dx$$
$$\int \frac{3}{x} dx + \int 1 dx$$

$$\int \frac{x+2}{x^2+1} dx = \int \frac{x}{x^2+1} dx + \int \frac{2}{x^2+1} dx$$

$$\int \tan x dx = \int \frac{\sin x}{\cos x} dx$$

$$\int \cot x dx = \int \frac{\cos x}{\sin x} dx$$

$$\int \frac{1}{\sec x} dx = \int \cos x dx$$

$$\int \frac{1}{\csc x} dx = \int \sin x dx$$

$$\int \frac{x^2-4}{x-2} dx = \int (x+2) dx$$

$$\int \frac{3x-2}{9x^2-4} dx = \int (3x+2) dx$$

$$\int \frac{x}{x^2} dx = \int \frac{1}{x} dx$$

Substitution Problems

$$\int (2x + 3)(x^2 + 3x + 9)^7 dx \quad u = x^2 + 3x + 9 \quad \text{New Integral } \int u^7 du$$
$$\frac{du}{dx} = 2x + 3 \quad \frac{u^8}{8} + C$$
$$du = (2x + 3)dx \quad \frac{1}{8}(x^2 + 3x + 9)^8 + C$$

$$2) \int (8x + 5)(4x^2 + 5x - 9)^6 dx \quad u = 4x^2 + 5x - 9 \quad \text{New Integral } \int u^6 du$$
$$\frac{du}{dx} = 8x + 5 \quad \frac{u^7}{7} + C$$
$$du = (8x + 5)dx \quad \frac{1}{7}(4x^2 + 5x - 9)^7 + C$$

$$3) \int \frac{6x-3}{3x^2-3x+9} dx \quad u = 3x^2 - 3x + 9 \quad \text{New Integral } \int \frac{du}{u}$$
$$\frac{du}{dx} = 6x - 3 \quad \ln|u| + C$$
$$du = (6x - 3)dx \quad \ln|3x^2 - 3x + 9| + C$$

$$4) \int \frac{2x+7}{x^2+7x-1} dx \quad u = x^2 + 7x - 1 \quad \text{New Integral } \int \frac{du}{u}$$
$$\frac{du}{dx} = 2x + 7 \quad \ln|u| + C$$
$$du = (2x + 7)dx \quad \ln|x^2 + 7x - 1| + C$$

$$5) \int (2x + 9)e^{x^2+9x-1} dx \quad u = x^2 + 9x - 1 \quad \text{New Integral } \int e^u du$$
$$\frac{du}{dx} = 2x + 9 \quad e^u + C$$
$$du = (2x + 9)dx \quad e^{x^2+9x-1} + C$$

$$6) \int (4x - 1)\sqrt{2x^2 - x} dx$$

$$u = 2x^2 - x$$

$$\int \sqrt{u} du$$

$$\frac{du}{dx} = 4x - 1$$

$$\frac{2}{3} u^{\frac{3}{2}} + C$$

$$du = (4x - 1)dx$$

$$\frac{2}{3} (2x^2 - x)^{\frac{3}{2}} + C$$

$$7) \int \frac{2x-5}{\sqrt{x^2-5x+8}} dx$$

$$u = x^2 - 5x + 8$$

$$\int \frac{du}{\sqrt{u}}$$

$$\frac{du}{dx} = 2x - 5$$

$$2u^{\frac{1}{2}} + C$$

$$du = (2x - 5)dx \quad 2(x^2 - 5x + 8)^{\frac{1}{2}} + C$$

$$8) \int \frac{2x-5}{(x^2-5x+4)^7} dx$$

$$u = (x^2 - 5x + 4)$$

$$\int \frac{du}{u^7}$$

$$\frac{du}{dx} = 2x - 5$$

$$-\frac{1}{6u^6} + C$$

$$du = (2x - 5)dx \quad -\frac{1}{6(x^2-5x+4)^6} + C$$

$$9) \int (2x + 5)\sin(x^2 + 5x - 2)dx$$

$$u = x^2 + 5x - 2$$

$$\int \sin u du$$

$$\frac{du}{dx} = 2x + 5$$

$$-\cos u + C$$

$$du = (2x + 5)dx \quad -\cos(x^2 + 5x - 2) + C$$

$$10) \int (4x - 1) \sec^2(2x^2 - x)dx$$

$$u = 2x^2 - x$$

$$\int \sec^2 u du$$

$$\frac{du}{dx} = 4x - 1$$

$$\tan u + C$$

$$du = (4x - 1)dx \quad \tan(2x^2 - x) + C$$

$$11) \int \cos x (\sin^5 x) dx$$

$$u = \sin x$$

$$\int u^5 du$$

$$\frac{du}{dx} = \cos x$$

$$\frac{1}{6} u^6 + C$$

$$du = \cos x dx$$

$$\frac{1}{6} (\sin^6 x) + C$$

$$12) \int (x + 5)(x^2 + 10x)^7 dx$$

$$u = x^2 + 10x$$

$$\frac{1}{2} \int u^7 du$$

$$\frac{du}{dx} = 2x + 10$$

$$\frac{1}{16} u^8 + C$$

$$du = (2x + 10) dx$$

$$\frac{1}{16} (x^2 + 10x)^8 + C$$

$$du = 2(x + 5) dx$$

$$\frac{du}{2} = (x + 5) dx$$

$$13) \int (x^2 - 2)e^{x^3 - 6x} dx$$

$$u = x^3 - 6x$$

$$\frac{1}{3} \int e^u du$$

$$\frac{du}{dx} = 3x^2 - 6$$

$$\frac{1}{3} e^u + C$$

$$du = (3x^2 - 6) dx$$

$$\frac{1}{3} e^{x^3 - 6x} + C$$

$$du = 3(x^2 - 2) dx$$

$$\frac{du}{3} = (x^2 - 2) dx$$

$$14) \int \frac{\sin(5x)}{\cos(5x)} dx$$

$$u = \cos 5x$$

$$-\frac{1}{5} \int \frac{du}{u}$$

$$\frac{du}{dx} = -5 \sin 5x$$

$$-\frac{1}{5} \ln|u| + C$$

$$du = (-5 \sin 5x) dx$$

$$-\frac{1}{5} \ln|\cos 5x| + C$$

$$\frac{du}{-5} = \sin 5x dx$$

$$15) \int \frac{2x + 2.25}{4x^2 + 9x - 2} dx$$

$$u = 4x^2 + 9x - 2 \quad \frac{1}{4} \int \frac{du}{u}$$

$$\frac{du}{dx} = 8x + 9 \quad \frac{1}{4} \ln|u| + C$$

$$du = (8x + 9)dx \quad \frac{1}{4} \ln|4x^2 + 9x - 2| + C$$

$$du = 4(x + 2.25)dx$$

$$\frac{du}{4} = (x + 2.25)dx$$

$$16) \int \frac{2x^3 + 3x + 6}{\sqrt{x^4 + 3x^2 + 12x + 2}} dx \quad u = x^4 + 3x^2 + 12x + 2$$

$$\frac{1}{2} \int \frac{du}{\sqrt{u}}$$

$$\frac{du}{dx} = 4x^3 + 6x + 12 \quad \frac{1}{2} (2)u^{\frac{1}{2}} + C$$

$$du = (4x^3 + 6x + 12)dx \quad u^{\frac{1}{2}} + C$$

$$du = 2(2x^3 + 3x + 6)dx \quad (x^4 + 3x^2 + 12x + 2)^{\frac{1}{2}} + C$$

$$\frac{du}{2} = (2x^3 + 3x + 6)dx$$

$$17) \int \frac{e^{\tan(3x)}}{\cos^2(3x)} dx$$

$$u = \tan 3x \quad \frac{1}{3} \int e^u du$$

$$\frac{du}{dx} = 3 \sec^2 3x \quad \frac{1}{3} e^u + C$$

$$du = 3 \sec^2 3x dx \quad \frac{1}{3} e^{\tan 3x} + C$$

$$du = \frac{3dx}{\cos^2 3x}$$

$$\frac{du}{3} = \frac{dx}{\cos^2 3x}$$

$$18) \int \frac{x+4}{x^2} dx$$

Divide and Conquer

$$\int \frac{x}{x^2} dx + \int \frac{4}{x^2} dx = \int \left(\frac{1}{x}\right) dx + \int \left(\frac{4}{x^2}\right) dx = \ln|x| - \frac{4}{x} + C$$

$$19) \int \frac{2x+3}{x^2+1} dx$$

Divide and Conquer

$$\int \frac{2x}{x^2+1} dx + \int \frac{3}{x^2+1} dx$$

$$\text{u-sub} \quad \tan^{-1} x$$

$$u = x^2 + 1$$

$$\frac{du}{dx} = 2x$$

$$du = 2x dx$$

$$\int \frac{du}{u} + 3 \tan^{-1} x + C$$

$$\ln|u| + 3 \tan^{-1} x + C$$

$$\ln|x^2 + 1| + 3 \tan^{-1} x + C$$

$$20) \int \frac{x+2}{x^2+4x+4} dx$$

$$\int \frac{x+2}{(x+2)^2} dx = \int \frac{1}{x+2} dx = \ln|x+2| + C$$

$$21) \int \frac{\ln x}{x} dx$$

$$u = \ln x$$

$$\int u du$$

$$\frac{du}{dx} = \frac{1}{x}$$

$$\frac{u^2}{2} + C$$

$$du = \frac{dx}{x}$$

$$\frac{1}{2} (\ln x)^2 + C$$

$$22) \int (x^2 + 3)^2 dx$$

You have to foil this one out.

$$\int (x^4 + 6x^2 + 9) dx$$

$$\frac{x^5}{5} + \frac{6x^3}{3} + 9x + C$$

$$\frac{x^5}{5} + 2x^3 + 9x + C$$

$$23) \int (x^2 + 2x + 5)^2 dx$$

You have to foil this out.

$$\int (x^2 + 2x + 5)(x^2 + 2x + 5) dx$$

$$\int (x^4 + 4x^3 + 14x^2 + 20x + 25) dx$$

$$\frac{1}{5}x^5 + x^4 + \frac{14}{3}x^3 + 10x^2 + 25x + C$$