Series on the BC MC Test

1) The Maclaurin series for $\frac{1}{1-x}$ is $\sum_{n=0}^{\infty} x^n$. What is a power series expansion for $\frac{x^2}{1-x^2}$?

$$x^2(1 + x^2 + x^4 + \cdots + x^{2n} + \cdots)$$

2) $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n!}$ is the Taylor series about zero for which function?

$$y = e^{-x}$$

3) A series expansion of $\frac{sint}{t}$ is

$$sint = t - \frac{t^3}{3!} - \frac{t^5}{5!} + \cdots + \frac{(-1)^{n+1} t^{2n+1}}{(2n+1)!}$$

$$\frac{sint}{t} = \frac{t}{t} - \frac{t^3}{3!\,t} - \frac{t^5}{5!\,t} + \cdots + \frac{(-1)^{n+1} t^{2n+1}}{(2n+1)!\,t}$$

$$\frac{sint}{t} = 1 - \frac{t^2}{3!} - \frac{t^4}{5!} + \cdots + \frac{(-1)^{n+1} t^{2n}}{(2n+1)!}$$

4) For $-1 < x < 1$ if $f(x) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^{2n-1}}{2n-1}$, then $f'(x) =$

$$\frac{(2n-1)(-1)^{n+1} x^{2n-2}}{2n-1} = \sum_{n=2}^{\infty} (-1)^{n+1} x^{2n-2}$$

5) The coefficient of x^3 in the Taylor series for e^{3x} about $x = 0$ is

$$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!}$$

$$e^{3x} = 1 + 3x + \frac{(3x)^2}{2} + \frac{(3x)^3}{3!} + \cdots + \frac{(3x)^n}{n!}$$

$$e^{3x} = 1 + 3x + \frac{9x^2}{2} + \frac{27x^3}{3!} + \cdots + \frac{(3x)^n}{n!}$$

$$\frac{27}{3!} = \frac{9}{2}$$

6) The Taylor series for $\sin(2x)$ is

$$sinx = x - \frac{x^3}{3!} - \frac{x^5}{5!} + \cdots + \frac{(-1)^{n+1} x^{2n+1}}{(2n+1)!}$$

$$sin2x = 2x - \frac{(2x)^3}{3!} - \frac{(2x)^5}{5!} + \cdots + \frac{(-1)^{n+1} (2x)^{2n+1}}{(2n+1)!}$$

7) The coefficient of x^6 in the Taylor's series expansion about $x = 0$ for $f(x) = \sin(x^2)$ is

$$sinx = x - \frac{x^3}{3!} - \frac{x^5}{5!} + \cdots + \frac{(-1)^{n+1}x^{2n+1}}{(2n+1)!}$$

$$sinx^2 = x^2 - \frac{(x^2)^3}{3!} - \frac{(x^2)^5}{5!} + \cdots + \frac{(-1)^{n+1}(x^2)^{2n+1}}{(2n+1)!}$$

$$-\frac{(x^2)^3}{3!} = -\frac{x^6}{6} = -\frac{1}{6}$$

8) (C) If $f(x) = \sum_{k=1}^{\infty}(sin^2x)^k$, then $f(1)$ is

Go Geo. $a_1 = sin^2(1) = .708 \quad r = .708 \quad \frac{a_1}{1-r} = \frac{.708}{1-.708} = 2.425$

9) The sum of the infinite geometric series $\frac{3}{2} + \frac{9}{16} + \frac{27}{128} + \frac{81}{1024} + \cdots$

Go Geo. $a_1 = \frac{3}{2} \quad r = \frac{3}{8} \quad \frac{a_1}{1-r} = \frac{\frac{3}{2}}{1-\frac{3}{8}} = \frac{\frac{3}{2}}{\frac{5}{8}} = \frac{3}{2}\cdot\frac{8}{5} = \frac{12}{5}$

10) Let f be the function given by $f(x) = \ln(3-x)$. The third degree Taylor polynomial for f about $x = 2$ is

$$\text{f(x)} = \ln(3-x) \qquad f(2) = ln1 = 0$$

$$f'(x) = \frac{-1}{3-x} = -(3-x)^{-1} \qquad f'(2) = \frac{-1}{1} = -1$$

$$f''(x) = -(3-x)^{-2} \qquad f''(2) = -\frac{1}{1} = -1$$

$$f'''(x) = -2(3-x)^{-3} \qquad f'''(2) = -2$$

$$f(2) + f'(2)(x-2) + f''(2)\frac{(x-2)^2}{2!} + f'''(2)\frac{(x-2)^3}{3!}$$

$$-(x-2) - \frac{(x-2)^2}{2!} - \frac{2(x-2)^3}{3!}$$

11) The Taylor series for $sinx$ about $x = 0$ is $x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots$. If f is a function such that $f'(x) = \sin(x^2)$, then the coefficient of x^7 in the Taylor series for $f(x)$ about $x = 0$ is

$$sinx = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots$$

$$f'(x) = sinx^2 = x^2 - \frac{(x^2)^3}{3} + \frac{(x^2)^5}{5} - \cdots$$

$$f(x) = \frac{x^3}{3} - \frac{x^7}{3(7)} + \frac{x^{11}}{11(5)}$$

The coefficient is $-\frac{1}{21}$.

12) What is the approximation of the value of sin 1 obtained by using the fifth-degree Taylor polynomial about $x = 0$ for $sinx$?

$$sinx = x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots$$

$$sin1 = 1 - \frac{1^3}{3} + \frac{1^5}{5} = 1 - \frac{1}{3} + \frac{1}{5} = \frac{15}{15} - \frac{5}{15} + \frac{3}{15} = \frac{13}{15}$$

13) If $\sum_{n=0}^{\infty} a_n x^n$ is a Taylor series that converges to $f(x)$ for all real x, then $f'(1) =$

$$f'(x) = a_n n x^{n-1} \qquad f'(1) = a_n n$$

14) (C) The graph of the function represented by the Maclaurin series

$1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots + \frac{(-1)^n x^n}{n!} + \cdots$ intersects the graph of $y = x^3$ at $x =$

$$1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots + \frac{(-1)^n x^n}{n!} + \cdots = e^{-x}$$

Put $y = x^3$& $y = e^{-x}$ and see they intersect at = .773 .

15) What is the value of $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n}$?

Go Geo. $\sum_{n=1}^{\infty} \frac{2^{n+1}}{3^n} = \sum_{n=1}^{\infty} \frac{2(2^n)}{3^n} = \sum_{n=1}^{\infty} 2(\frac{2}{3})^n$

$a_1 = \frac{4}{3}$ $r = \frac{2}{3}$ $\frac{\frac{4}{3}}{1-\frac{2}{3}} = \frac{\frac{4}{3}}{\frac{1}{3}} = 4$

16) A function f has Maclaurin series given by $\frac{x^4}{2!} + \frac{x^5}{3!} + \frac{x^6}{4!} + \cdots + \frac{x^{n+3}}{(n+1)!} + \ldots$

Which of the following is an expression for (x) ? D

a) $-3xsinx + 3x^2$
b) $-\cos(x^2) + 1$
c) $-x^2cosx + x^2$
d) $x^2e^x - x^3 - x^2$
e) $e^{x^2} - x^2 - 1$

$$x^2e^x - x^3 - x^2 = x^2\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right) - x^3 - x^2$$
$$x^2 + x^3 + \frac{x^4}{2!} + \frac{x^5}{3!} + \cdots - x^3 - x^2$$
$$= \frac{x^4}{2!} + \frac{x^5}{3!} + \cdots$$

17) What is the coefficient of x^2 in the Taylor series for $\frac{1}{1+x^2}$ about $x = 0$?

Go Geo. $r = -x^2$

$$\frac{1}{1+x^2} = 1 - x^2 + \frac{(-x^2)^2}{2!} + ..$$

The coefficient is -1.

18) Let $P(x) = 3x^2 - 5x^3 + 7x^4 + 3x^5$ be the fifth-degree Taylor polynomial for the function f about $x = 0$. What is the value of $f'''(0)$?

$$P(x) = 3x^2 - 5x^3 + 7x^4 + 3x^5$$

$$P(x) = f(0) + f'(0)x + f''(0)\frac{x^2}{2!} + f'''(0)\frac{x^3}{3!} + f''''(0)\frac{x^4}{4!} + f'''''(0)\frac{x^5}{5!}$$

$$f'''(0)\frac{x^3}{3!} = -5x^3$$

$$\frac{f'''(0)}{3!} = -5$$

$$f'''(0) = -5(3!) = -30$$

19) What is the sum of the series $1 + ln2 + \frac{(ln2)^2}{2!} + \cdots + \frac{(ln2)^n}{n!} + \cdots$

This is $e^x = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!}$ Where the $x = ln2$.

$$e^{ln2} = 2$$

20) If $f(x) = xsin(2x)$, what is the Taylor series for f about $x = 0$?

$$sinx = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

$$sin2x = 2x - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} - \frac{(2x)^7}{7!} + \cdots = 2x - \frac{8x^3}{3!} + \frac{32x^5}{5!} - \frac{128x^7}{7!}$$

$$xsin2x = 2x^2 - \frac{8x^4}{3!} + \frac{32x^6}{5!} - \frac{128x^8}{7!}$$