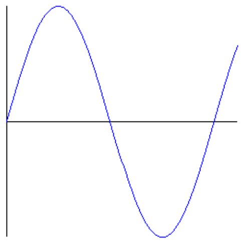
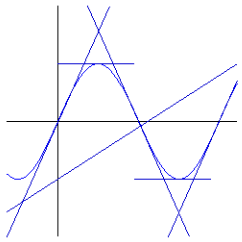
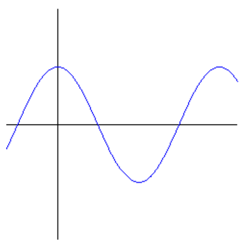
Calculus 3.5  Trigonometric Functions

Graph $y = sinx$ using five key points.  Draw tangent lines at each of the five points.





Graph the derivative of $y = sinx$ by finding the derivative at the 5 basic points.

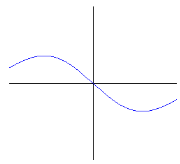


The derivative of $y = sinx$  is $y' = cosx$

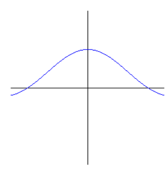
Prove your answer using the definition of the derivative.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\sin(x+h)-sinx}{h}=\lim_{h\to 0}\frac{sinxcosh+cosxsinh-sinx}{h}$$

$$\lim_{h\to 0}\frac{sinxcosh-sinx}{h}+\lim_{h\to 0}\frac{cosxsinh}{h}=sinx\lim_{h\to 0}\frac{cosh-1}{h}+cosx\lim_{h\to 0}\frac{sinx}{x}$$



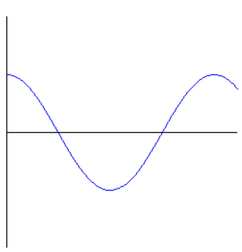
$\lim_{h\to 0}\frac{cosh-1}{h}=0$ because of this graph.

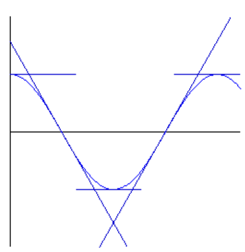


$\lim_{h\to 0}\frac{sinx}{x}=1$ because of this graph

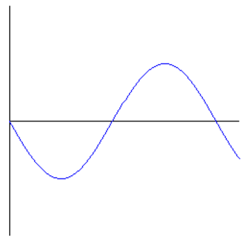
$$sinx(0)+cosx(1)=cosx$$

Graph $y=cosx$ showing the key five points and draw tangents at each of the 5 points.





Graph the derivative of $y = cosx$ by finding the derivative at the 4 basic points.



The derivative of $y = cosx$ is $-sinx$.

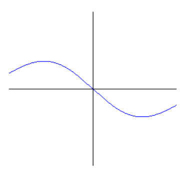
Prove your answer using the definition of the derivative.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{cosxcosh - sinxsinh - cosx}{h}$$

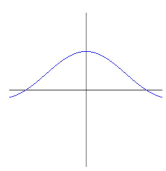
$$= \lim_{h \to 0} \frac{cosxcosh - cosx}{h} - \lim_{h \to 0} \frac{sinxsinh}{h}$$

$$\lim_{h \to 0} \frac{cosx(cosh - 1)}{h} - \lim_{h \to 0} \frac{sinxsinh}{h} =$$

$$cosx\lim_{h \to 0} \frac{(cosh-1)}{h} - sinx\lim_{h \to 0} \frac{sinh}{h} =$$



$\lim_{h \to 0} \frac{(cosh-1)}{h} = 0$ because of the graph



$\lim_{h \to 0} \frac{sinh}{h} = 1$ because of the graph

$cosx(0) - sinx(1) = -sinx$

Find the derivative of $y = tanx$ by using a trig identity.

$$y = \frac{sinx}{cosx}$$

Use the quotient rule to find the derivative.

$$\frac{BDT - TDB}{B^2} = \frac{cosx(cosx) - sinx(-sinx)}{cos^2x} = \frac{cos^2x + sin^2x}{cos^2x} = \frac{1}{cos^2x} = sec^2x$$

Find the derivative of $y = cotx$ by using a trig identity.

$$y = \frac{cosx}{sinx}$$

Use the quotient rule to find the derivative.

$$\frac{BDT - TDB}{B^2} = \frac{sinx(-sinx) - cosx(-cosx)}{cos^2x} = \frac{-sin^2x - cos^2x}{sin^2x} =$$

$$\frac{-(sin^2x + cos^2x)}{sin^2x} = \frac{-1}{sin^2x} = -csc^2x$$

Find the derivative of $y = secx$ by using a trig identity.

$$y = \frac{1}{cosx}$$

Use the quotient rule to find the derivative.

$$\frac{BDT - TDB}{B^2} = \frac{cosx(0) - 1(-sinx)}{cos^2x} = \frac{sinx}{cos^2x} = \frac{sinx}{cosx} \cdot \frac{1}{cosx} = tanxsecx = secxtanx$$

Find the derivative of $y = cscx$ by using a trig identity.

$$y = \frac{1}{sinx}$$

Use the quotient rule to find the derivative.

$$\frac{BDT - TDB}{B^2} = \frac{sinx(0) - 1(cosx)}{sin^2x} = \frac{-cosx}{sin^2x} = \frac{-cosx}{sinx} \cdot \frac{1}{sinx} = -cotxcscx = -cscxcot$$

| Function | Derivative |
|---|---|
| $y = sinx$ | $y' = cosx$ |
| $y = cosx$ | $y' = -sinx$ |
| $y = tanx$ | $y' = sec^2x$ |
| $y = cotx$ | $y' = -csc^2x$ |
| $y = secx$ | $y' = secxtanx$ |
| $y = cscx$ | $y' = -cscxcotx$ |

All of the cofunctions are negative.

$y = x^2tanx$ Product Rule $x^2$ $tanx$

$2x$ $sec^2x$

$y' = x^2sec^2x + 2xtanx$

$y = \frac{sinx}{3x^2+2x}$ $sinx$ $3x^2 + 2x$

$cosx$ $6x + 2$

$y' = \frac{BDT-TDB}{B^2} = \frac{(3x^2+2x)cosx-(6x+2)sinx}{(3x^2+2x)^2}$

$s(t) = 2sint + 3cost$ Find the velocity, speed and acceleration at $t$.

$v(t) = 2cost - 3sint$

$speed(t) = |2cost - 3sint|$

$a(t) = -2sint - 3cost$

Find the position, velocity, speed and acceleration at $t = \frac{\pi}{4}$.

$s(t) = 2\frac{\sqrt{2}}{2} + 3\frac{\sqrt{2}}{2} = \frac{5\sqrt{2}}{2}$

$v(t) = 2\frac{\sqrt{2}}{2} - 3\frac{\sqrt{2}}{2} = \frac{-\sqrt{2}}{2}$

$speed(t) = \left|2\frac{\sqrt{2}}{2} - 3\frac{\sqrt{2}}{2}\right| = \frac{\sqrt{2}}{2}$

$a(t) = -2\frac{\sqrt{2}}{2} - 3\frac{\sqrt{2}}{2} = -\frac{5\sqrt{2}}{2}$

Find the tangent and normal line of $y = secx$ at $x = \frac{\pi}{3}$.

$x = \frac{\pi}{3}$ $y = sec\frac{\pi}{3} = 2$ $y' = secxtanx = 2\sqrt{3}$

$y - 2 = 2\sqrt{3}(x - \frac{\pi}{3})$ tangent line

$y - 2 = -\frac{1}{2\sqrt{3}}(x - \frac{\pi}{3})$ normal line

Find the equation of the normal and tangent line at $x = 3$ for $y = x^2 sinx$.

$y' = x^2cosx + 2xsinx$

$y' = 3^2cos3 + 2(3)sin3 = -8.063$

$x = 3, y = 1.27$

$y - 1.27 = -8.063(x - 3)$ Tangent line

$y - 1.27 = .124(x - 3)$ Normal line

Find the equation for the horizontal tangent over the interval $[0, \pi]$ for $y = 2sinx$.

The horizontal tangent has a slope of 0.

Find the derivative and make it = 0. Solve for $x$.

$y' = 2cosx \qquad 0 = 2cosx \qquad 0 = cosx \qquad x = \frac{\pi}{2}$

$y = 2\sin\left(\frac{\pi}{2}\right) = 2(1) = 2$

The point is $(\frac{\pi}{2}, 2)$ and the slope is 0.

$y - 2 = 0(x - \frac{\pi}{2})$

$y - 2 = 0$

$y = 2$