Calculus 3.4 Notes

A position graph will give the location of an object at a certain time.

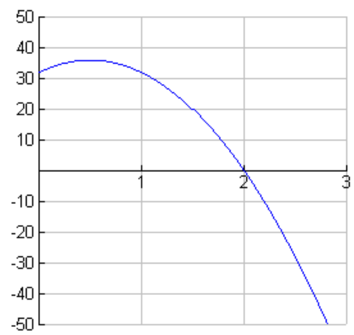
At t = 4, the car is 20 miles away from where it started.

A position function is usually written as $s(t)$ or $x(t)$.

If the position is a positive number, the object is either right or up from the starting location.

If the position is a negative number, the object is either left or down from the starting position.

A diver jumps from a platform diving board that is 32 feet above the water. The position of the diver is given by $s(t) = -16t^2 + 16t + 32$. The s is in feet and the t is in seconds.



What is the location at $t = 1$? 32 $t = 0$? 32 $t = 2$? 0

What is the highest location? 36 When does that occur? At $t = \frac{1}{2}$

When does the diver hit the water? At $t = 2$.

What is the graph showing after $t = 2$? The graph is pretending that the diver would continue into the water and into the ground below the pool at the same rate.

When is the diver going up? The diver is going up from $t = 0 \rightarrow t = \frac{1}{2}$.

When is the diver going down? The diver is going down from $t = \frac{1}{2} \rightarrow t = 2$.

The velocity graph is showing the change in position in terms of time.

At $t = 4$, the car is moving at $25\,\frac{m}{hr}$.

Speed = $|velocity|$.

If we wanted the average change in position (the slope) for the first 2 seconds, we would find the position at $t = 0$ and the position at $t = 2$, then find the slope. $s(t) = -16t^2 + 16t + 32$

$t = 0 \rightarrow s(0) = 32$ $\quad\quad$ $t = 2 \rightarrow s(2) = 0$.

The average change in position (slope) is $\frac{0-32}{2-0} = -16\,\frac{ft}{sec}$.

Find the AROC over the first second.

$t = 0 \rightarrow s(0) = 32$ $\quad\quad$ $t = 1 \rightarrow s(1) = 32$.

The average change in position (slope) is $\frac{32-32}{1-0} = 0\,\frac{ft}{sec}$.

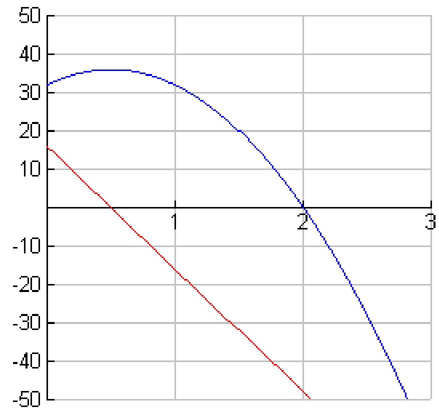
Does this tell you what the diver is doing?

It really does not describe the diver going up or down.

To find the diver's velocity at one specific time, we take the derivative.

$s(t) = -16t^2 + 16t + 32$

$v(t) = -32t + 16$



From $t = 0$ to $t = \frac{1}{2}$, what is the diver doing?

If the diver is going up, the velocity must be going up.

Is the graph of the velocity positive from $t = 0$ to $t = \frac{1}{2}$?   Yes.

From $t = \frac{1}{2}$ to $t = 2$, what is the diver doing?

If the diver is going down, the velocity must be negative.

Is the graph of the velocity negative from $t = \frac{1}{2}$ to $= 2$? Yes.

To find the average acceleration over the first two seconds, we take the velocity at $t = 0$ and $t = 2$,

and find the slope. $t = 0 \rightarrow v = 16$ $t = 2 \rightarrow v = -48$

$\frac{16+48}{0-2} = \frac{64}{-2} = -32ft/sec$

What would the average acceleration be over any two points that I pick? -32 ft/sec

Acceleration is the change in velocity in terms of time.

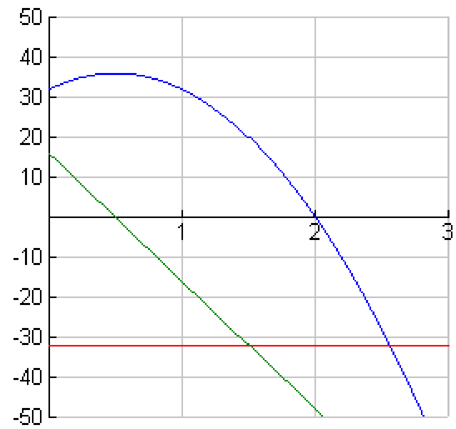
To find the acceleration at a certain time, we find the derivative of the velocity.

$$a(t) = v'(t)$$

Since the velocity is the change in position, the acceleration is the change of the change in position.

$$a(t) = v'(t) = s''(t)$$

$$a'(t) = -32$$



From $t = 0\ to\ t = \frac{1}{2}$

The position is positive.

The velocity is positive.

The speed is positive.

The acceleration is negative.

The diver is speeding up, slowing down or staying constant?   Slowing down.

From $t = \frac{1}{2}\ to\ t = 2$

The position is positive.

The velocity is negative.

The acceleration is negative.

The diver is speeding up, slowing down or staying constant? Speeding up.

From $t = 2\ to\ t = 3$

The position is negative.

The velocity is negative.

The acceleration is negative.

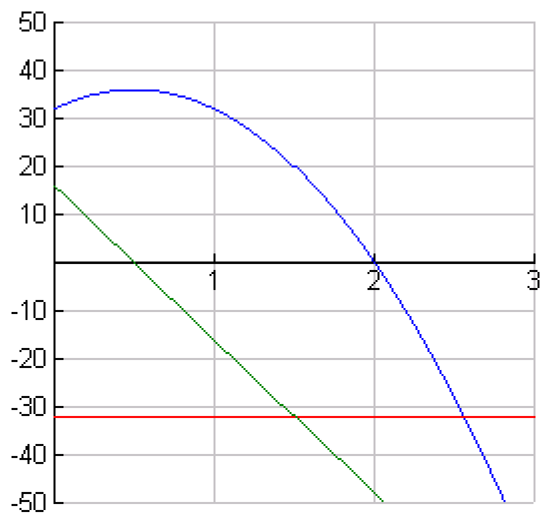
The diver is speeding up, slowing down or staying constant? Speeding up.

Positive velocity with negative acceleration makes slowing down to the right.

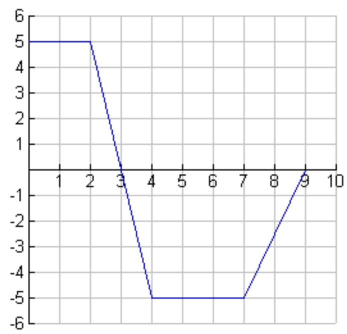
Negative velocity with negative acceleration makes speeding up to the left.

Positive velocity with positive acceleration makes speeding up to the right.

Negative velocity with positive acceleration makes slowing down to the left.



This is a velocity graph. The y-coordinate represents the velocity = $\frac{\Delta position}{\Delta time}$ and the x-coordinate represents the time.



Positive velocity means that the object is moving to the right or up.

Negative velocity means that the object is moving to the left or down.

When the velocity is 0, the object has stopped.

When the velocity changes from + to – OR from – to +, the object has changed direction.

The speed is the absolute value of the velocity.

The slope or derivative of the velocity curve is the acceleration.

When the slope = acceleration = 0, the object is traveling at a constant speed.

When $a > 0$ & $v > 0$, the object is speeding up to the right.

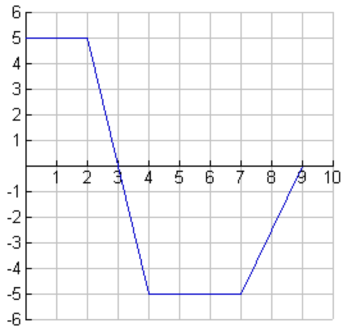
When $a > 0$ & $v < 0$, the object is slowing down to the left.

When $a < 0$ & $v > 0$, the object is slowing down to the right.

When $a < 0$ & $v < 0$, the object is speeding up to the left.

Quick trick on slowing down or speeding up:

$av > 0$ speeding up $\qquad av <$ slowing down



From $t = 0 \rightarrow t = 2$

The velocity is positive.

The acceleration is 0.

The object is traveling to the right.

Is the object speeding up, slowing down or at a constant rate? Constant rate.

From $t = 2 \rightarrow t = 3$

The velocity is positive.

The acceleration is negative.

The object is traveling to the right.

Is the object speeding up, slowing down or at a constant rate? Slowing down.

From $t = 3 \rightarrow t = 4$

The velocity is negative.

The acceleration is negative.

The object is traveling to the left.

Is the object speeding up, slowing down or at a constant rate? Speeding up.

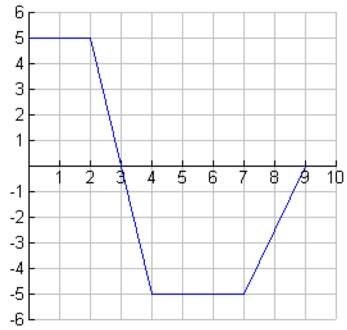
From $t = 4 \rightarrow t = 7$

The velocity is negative.

The acceleration is 0.

The object is traveling to the left.

Is the object speeding up, slowing down or at a constant rate? At a constant rate.



From $t = 7 \to t = 9$

The velocity is negative.

The acceleration is positive.

The object is traveling to the left.

Is the object speeding up, slowing down or at a constant rate? Slowing down.

When does the object change direction?

At $t = 3$, is the object to the right or left of the starting point? The object has just traveled to the right.

At $t = 4$, is the object to the right or left of the starting point? The object has traveled to the right for a long time and to the left for just a little time.

At $t = 7$, is the object to the right or left of the starting point? The object has traveled more to the left than to the right.

At $t = 9$, is the object to the right or left of the starting point? The object has continued to travel to the left..

When is the object at the starting point?

At t = 0 and t = 6. At the start and when the distance traveled to the left equals the distance traveled to the right.

A position of a particle is described by the equation $s(t) = 3t^2 - 3t$. The distance is in feet and the time is in seconds. Find the total distance traveled over the first 6 seconds.

The first question that needs to be asked is: Does the particle change direction?

The particle can only change direction if the velocity equals zero.

The velocity is $v(t) = s'(t) = 6t - 3$.

The velocity equals zero when $0 = 6t - 3$ or when $t = \frac{1}{2}$.

Which direction is the particle moving from $t = 0$ to $t = \frac{1}{2}$?

$s(0) = 3(0)^2 - 3(0) = 0$

$s\left(\frac{1}{2}\right) = 3(\frac{1}{2})^2 - 3\left(\frac{1}{2}\right) = \frac{3}{4} - \frac{3}{2} = -\frac{3}{4}$.

The position started at 0 and moved to $-\frac{3}{4}$. It traveled $\frac{3}{4}$ feet.

Which direction is the particle moving from $t = \frac{1}{2}$ to $t = 6$?

$s\left(\frac{1}{2}\right) = -\frac{3}{4}$

$s(6) = 3(6)^2 - 3(6) = 108 - 18 = 90$.

The particle started at $-\frac{3}{4}$ and ended at 90. It traveled $90\frac{3}{4}$ feet.

The first $\frac{1}{2}$ second it traveled ¾ feet and the next $5\frac{1}{2}$ seconds it traveled $90\frac{3}{4}$ feet for a total distance

of $91\frac{1}{2}$ feet.

Displacement is defined as the net distance from the start to the finish.

The particle started at 0 and ended at 90. the displacement was 90 feet.