Calculus 3.3 The Shortcuts

The POWER rule:

$y = x^5$ $y' = 5x^4$

$y = 3x^6$ $y' = 18x^5$

$y = 7x$ $y' = 7$

$y = 8$ $y' = 0$

$y = \frac{1}{x^3}$ Change it to $y = x^{-2}$ $y' = -2x^{-3} = -\frac{2}{x^3}$

$y = \frac{8}{x^5}$ Change it to $y = -8x^{-5}$ $y' = 40x^{-6} = \frac{40}{x^6}$

$y = \sqrt{x}$ Change it to $y = x^{\frac{1}{2}}$ $y' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$y = \sqrt[3]{x}$ Change it to $y = x^{\frac{1}{3}}$ $y' = \frac{1}{3}x^{-\frac{2}{3}} = \frac{1}{3\sqrt[3]{x^2}}$

What is the general power rule?

$y = ax^n$ $y' = anx^{n-1}$

Two functions added or subtracted together

$y = x^5 + 7x^2 + 9x - 10$ $y' = 5x^4 + 14x + 9$

$y = \sqrt{x} - \frac{3}{x}$ Change it to $y = x^{\frac{1}{2}} - 3x^{-1}$ $y' = \frac{1}{2}x^{-\frac{1}{2}} + 3x^{-2} = \frac{1}{2\sqrt{x}} + \frac{3}{x^2}$

What is the general rule for two functions added or subtracted together ?

$y = f(x) + g(x)$ $y' = f'(x) + g'(x)$

The Product Rule

$y = (x+5)(7x-1)$ There are two 2 ways to solve this problem.

Method 1:

Multiply the functions together and then take the derivative.

$y = 7x^2 + 34x - 5$ $\qquad y' = 14x + 11$

Method 2

Use the Product Rule.

Identify the two functions and name them.

$f(x) = x + 5$ $\qquad g(x) = 7x - 1$

Take the derivative of each function.

$f'(x) = 1$ $\qquad g'(x) = 7$

Use formula $y' = f'(x)g(x) + f(x)g'(x)$

$y' = 1(7x-1) + (x+5)(7)$

$y' = 7x - 1 + 7x + 35 = 14x + 34$

Practice

$$y = (7x-3)(\sqrt{x+8})$$

$f(x) = 7x - 3$ $\qquad$ $g(x) = \sqrt{x+8}$

$f'(x) = 7$ $\qquad$ $g(x) = (x+8)^{\frac{1}{2}}$

$g'(x) = \frac{1}{2}(x+8)^{-\frac{1}{2}}$

$g'(x) = \frac{1}{2\sqrt{x+8}}$

Using the formula $y' = f'(x)g(x) + f(x)g'(x)$ is the same thing as cross multiplying (do the arrows) and adding the answers together.

Use formula $y' = \frac{(7x-3)}{2\sqrt{x+8}} + 7\sqrt{x+8}$

Add these together using the diagonal - diagonal – denominator trick.

$$y' = \frac{(7x-3)}{2\sqrt{x+8}} + \frac{7\sqrt{x+8}}{1} = \frac{7x-3+14(x+8)}{2\sqrt{x+8}} = \frac{21x+109}{2\sqrt{x+8}}$$

More Practice

$$y = (6x^2 - 9x)(\sqrt[3]{x})$$

$f(x) = 6x^2 - 9x$ $\qquad$ $g(x) = \sqrt[3]{x}$

$f'(x) = 12x - 9$ $\qquad$ $g'(x) = \frac{1}{3}x^{-\frac{2}{3}}$

$g'(x) = \frac{1}{3\sqrt[3]{x^2}}$

Do the arrows.

$$y' = \frac{6x^2-9x}{3\sqrt[3]{x^2}} + (12x-9)\sqrt[3]{x}$$

Add these together using the diagonal - diagonal – denominator trick.

$$y' = \frac{6x^2-9x+(36x-27)x}{3\sqrt[3]{x^2}} = \frac{45x^2-36x}{3\sqrt[3]{x^2}} = \frac{15x^2-12x}{\sqrt[3]{x^2}}$$

The Quotient Rule

When you see a function divided by another function, you use the quotient rule.

$y = \frac{4x+7}{3x-2}$ Split the fraction into a Top function and a Bottom function.

Top = $4x + 7$ Bottom = $3x - 2$ Now take the derivative of both of them.

Top′ = 4 Bottom ′ = 3

Use this formula:

$$\frac{(bottom \cdot the\ derivative\ of\ the\ top) - (top \cdot the\ derivative\ of\ the\ bottom)}{(bottom)^2}$$

$$\frac{(3x-2)(4)-(4x+7)(3)}{(3x-2)^2} = \frac{12x-8-12x-21}{(3x-2)^2} = \frac{-29}{(3x-2)^2}$$

Try it again

$y = \frac{\sqrt{x}-1}{\sqrt{x}+1}$ top = $\sqrt{x} - 1$ bot = $\sqrt{x} + 1$

Top ′ $= \frac{1}{2\sqrt{x}}$ bot ′ $= \frac{1}{2\sqrt{x}}$

$$y' = \frac{bdt-tdb}{b^2} = \frac{\frac{\sqrt{x}+1}{2\sqrt{x}} - \frac{\sqrt{x}-1}{2\sqrt{x}}}{(\sqrt{x}+1)^2} = \frac{\frac{\sqrt{x}+1-\sqrt{s}+1}{2\sqrt{x}}}{(\sqrt{x}+1)^2} = \frac{2}{2\sqrt{x}(\sqrt{x}+1)^2} = \frac{1}{\sqrt{x}(\sqrt{x}+1)^2}$$