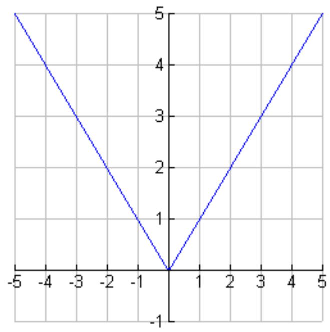
3.2 Notes

There are four different types of functions that do not have derivatives at certain points.

Graph $y = |x|$.

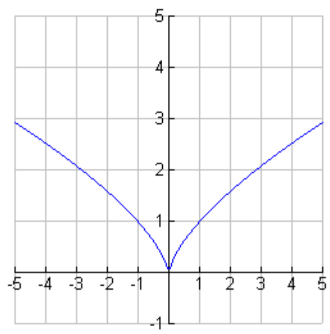


If $x < 0$, the derivative is -1. If $x > 0$, the derivative is 1.

What is the derivative at $x = 0$? Since the left derivative does not equal the right derivative, the derivative does not exist.

This is called a corner.

Graph $y = \sqrt[3]{x^2}$.

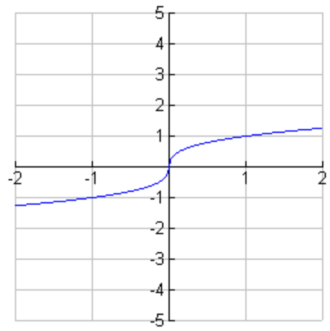


What is the derivative when $x < 0$? Negative What is the derivative when $x > 0$? Positive

What is the derivative when $x = 0$? Since the left derivative does not equal the right derivative, the derivative does not exist.

This is called a cusp.

Graph $y = \sqrt[3]{x}$

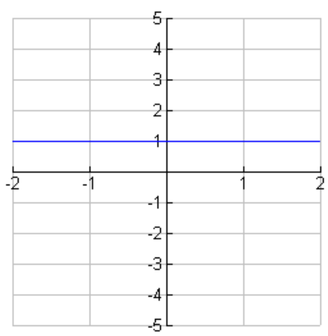


What is the derivative when $x < 0$? Positive What is the derivative when $x > 0$? Positive
When $x = 0$, the derivative looks like a vertical line which is undefined.

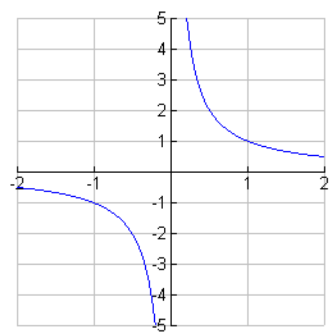
This is called a vertical tangent.

Graph any type of discontinuity.

$y = \frac{x-2}{x-2}$



$y = \frac{1}{x}$



It should make sense that if there is value for an x, there is no derivative for the x.

These are called discontinuities.

The four types of functions that are not differentiable are:

1) Corners
2) Cusps
3) Vertical tangents
4) Any discontinuities

Give me a function is that is continuous at a point but not differentiable at the point.

A graph with a corner would do.

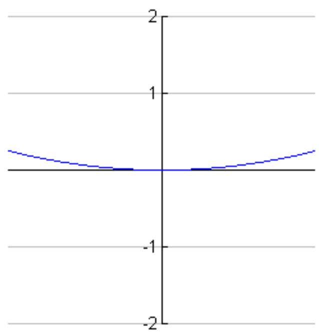
Give me a graph that is differentiable at a point but not continuous at the point.

You can't do it. If it is differentiable at a point, it must be continuous at the point.

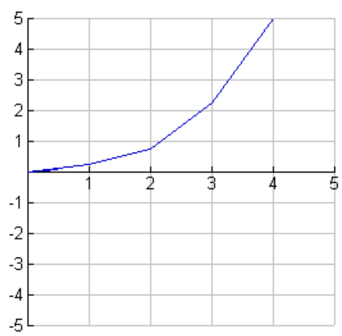
Differentiability implies continuity.

Continuity does not imply differentiability.

If you take any curve and look at it over a very small x interval, it looks like a line.



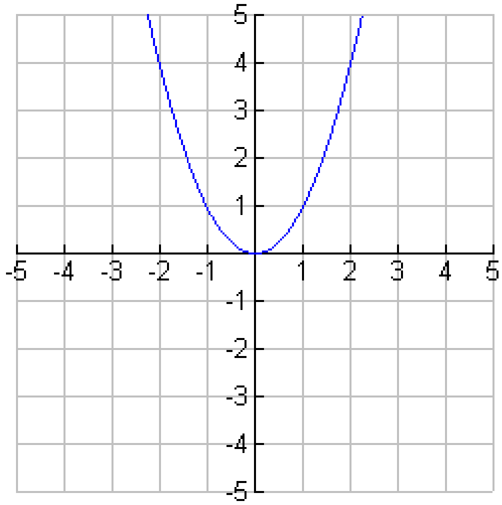
This is the graph of $y = x^2$ over the x interval [0, 4] that has been decurved (my word).



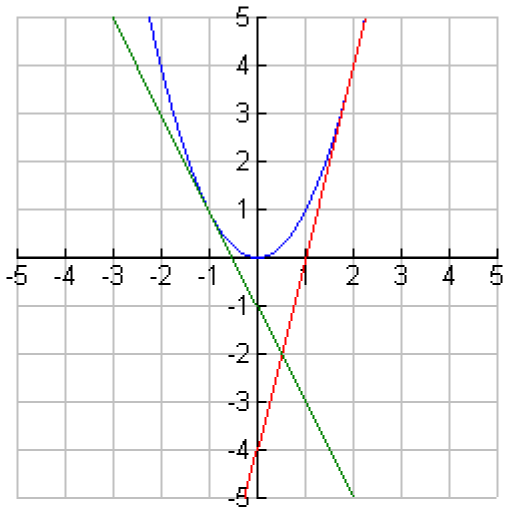
I have made little line segments with different slopes to simulate the curve.

This is called linearization.

Let’s look at the curve $y = x^2$. What is the derivative? $y' = 2x$.



I have created two tangent lines.
One line goes through (-1, 1) and the other goes through (2, 4).



I have created two tangent lines.
One line goes through (-1, 1) and the other goes through (2, 4).

What is the slope at $x = -1$?
$f'(-1) = -2$

What is the slope at $x = 2$?
$f'(2) = 4$

Between $f'(-1)$ & $f'(2)$, we can find every number.

There is some x between -1 & 2 that will create every derivative between -2 & 4.

This is called the Intermediate Value for Derivatives.

What do you know if $\lim_{x \to 3} f(x) = 8$? Draw a graph to disprove it if you can.

Is $f(x)$ continuous at $x = 3$? No. There could be a hole.

Is $f(x)$ differentiable at $x = 3$? If it is differentiable, it must be continuous, but it might not be.

Is $f(3) = 8$? No, if there is a piecewise function, the value might not equal the limit.

Does $\lim_{x \to 3^-} f(x) = 8$? If the limit exists, the left side limit must equal the right side limit.

Does $\lim_{x \to 3^+} f(x) = 8$? Same as above.

What do you know if $\lim_{x \to 1} \frac{3x^2 - 3}{x - 1} = 6$?

This is the definition of the derivative in the form of $\lim_{x \to a} \frac{f(x) - f(a)}{x - a} = f'(a)$

Is $f'(1) = 6$? Yes, by this definition.

Is $f(x)$ differentiable at $x = 1$? Yes, differentiability implies continuity.

Is $f(1) = 6$? We know nothing about the value at $x = 1$.

Does $\lim_{x \to 1^-} f(x) = 6$? We know nothing about the limit at $x = 1$.

Does $\lim_{x \to 1^+} f(x) = 6$? We know nothing about the limit at $x = 1$.