Calculus 3.1

There are three different formulas to find a derivative.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \qquad \lim_{x\to a}\frac{f(x)-f(a)}{x-a} \qquad \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

Find $f'(3)$ when $f(x) = x^2 + 7x - 4$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} \qquad \lim_{x\to a}\frac{f(x)-f(a)}{x-a} \qquad \lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$$

$\lim_{h\to 0}\frac{(x+h)^2+7(x+h)-4-(x^2+7x-4)}{h}$ $\qquad$ $\lim_{x\to 3}\frac{x^2+7x-4-26}{x-3}$ $\qquad$ $\lim_{h\to 0}\frac{(3+h)^2+7(3+h)-4-26}{h}$

$\lim_{h\to 0}\frac{x^2+2xh+h^2+7x+7h-4-x^2-7x+4}{h}$ $\qquad$ $\lim_{x\to 3}\frac{x^2+7x-30}{x-3}$ $\qquad$ $\lim_{h\to 0}\frac{9+6h+h^2+21+7h-30}{h}$

$\lim_{h\to 0}\frac{2xh+h^2+7h}{h}$ $\qquad$ $\lim_{x\to 3}\frac{(x+10)(x-3)}{x-3}$ $\qquad$ $\lim_{h\to 0}\frac{h^2+13h}{h}$

$\lim_{h\to 0}\frac{h(2x+h+7)}{h}$ $\qquad$ $\lim_{x\to 3}(x+10)$ $\qquad$ $\lim_{h\to 0}\frac{h(h+13)}{h}$

$\lim_{h\to 0}(2x+h+7)$ $\qquad$ 13 $\qquad$ $\lim_{h\to 0}(h+13)$

$2x+7$ $\qquad$ 13

At $x = 3, f'(3) = 13$

The first method, finds the derivative at any point. The second two methods create a derivative at a certain point.

Find the derivative of $f(x) = \sqrt{4x+1}$ at $x = 2$ using all three methods.

$$\lim_{h\to 0}\frac{\sqrt{4(x+h)+1}-\sqrt{4x+1}}{h}$$

$$\lim_{h\to 0}\frac{\sqrt{4(x+h)+1}-\sqrt{4x+1}}{h}\,\frac{\sqrt{4(x+h)+1}+\sqrt{4x+1}}{\sqrt{4(x+h)+1}+\sqrt{4x+1}}$$

$$\lim_{h\to 0}\frac{4(x+h)+1-(4x+1)}{h(\sqrt{4(x+h)+1}+\sqrt{4x+1})} = \lim_{h\to 0}\frac{4h}{h(\sqrt{4(x+h)+1}+\sqrt{4x+1})}$$

$\lim_{h\to 0}\frac{4}{\sqrt{4(x+h)+1}+\sqrt{4x+1}} = \frac{4}{2\sqrt{4x+1}}$ when x = 2 $\quad \frac{4}{2\sqrt{4(2)+1}} = \frac{4}{6} = \frac{2}{3}$

$$\lim_{x\to 2}\frac{\sqrt{4x+1}-3}{x-2}$$

$$\lim_{h\to 0}\frac{\sqrt{4(2+h)+1}-3}{h}$$

$$\lim_{x\to 2}\frac{\sqrt{4x+1}-3}{x-2}\ \frac{\sqrt{4x+1}+3}{\sqrt{4x+1}+3}$$

$$\lim_{h\to 0}\frac{\sqrt{4(2+h)+1}-3}{h}\cdot\frac{\sqrt{4(2+h)+1}+3}{\sqrt{4(2+h)+1}+3}$$

$$\lim_{x\to 2}\frac{4x+1-9}{(x-2)(\sqrt{4x+1}+3)}=$$

$$\lim_{h\to 0}\frac{4(2+h)+1-9}{h(\sqrt{4(2+h)+1}+3)}$$

$$\lim_{x\to 2}\frac{4x-8}{(x-2)(\sqrt{4x+1}+3)}=$$

$$\lim_{h\to 0}\frac{8+4h+1-9}{h(\sqrt{4(2+h)+1}+3)}$$

$$\lim_{x\to 2}\frac{4(x-2)}{(x-2)(\sqrt{4x+1}+3)}=$$

$$\lim_{h\to 0}\frac{4h}{h(\sqrt{4(2+h)+1}+3)}$$

$$\lim_{x\to 2}\frac{4}{(\sqrt{4x+1}+3)}=\frac{4}{6}=\frac{2}{3}$$

$$\lim_{h\to 0}\frac{4}{\sqrt{4(2+h)+1}+3}=\frac{4}{6}=\frac{2}{3}$$

All three formulas give the same answer.

Find the derivative of $f(x)=\frac{x+4}{x-2}$ at $x=1$using all three methods.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$\lim_{h\to 0}\frac{\frac{x+h+4}{x+h-2}-\frac{x+4}{x-2}}{h}=\lim_{h\to 0}\frac{\frac{(x+h+4)(x-2)-(x+h-2)(x+4)}{(x+h-2)(x-2)}}{h}$$

$$\lim_{h\to 0}\frac{\frac{x^2+xh+4x-2x-2h-8-x^2-4x-xh-4h+2x+8}{(x+h-2)(x-2)}}{h}=\lim_{h\to 0}\frac{-6h}{h(x+h-2)(x-2)}=\lim_{h\to 0}\frac{-6}{(x+h-2)(x-2)}$$

$\frac{-6}{(x-2)^2}$ at $x=1$ is $\frac{-6}{1}=-6$

$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a} \qquad f(1)=-5$$

$$\lim_{x\to 1}\frac{\frac{x+4}{x-2}+5}{x-1}=\lim_{x\to 1}\frac{\frac{x+4+5x-10}{x-2}}{x-1}=\lim_{x\to 1}\frac{\frac{6x-6}{x-2}}{x-1}=\lim_{x\to 1}\frac{6x-6}{(x-2)(x-1)}$$

$$\lim_{x\to 1}\frac{6(x-1)}{(x-2)(x-1)}=\lim_{x\to 1}\frac{6}{x-2}=\frac{6}{-1}=-6$$

$$\lim_{h\to 0}\frac{f(a+h)-f(a)}{h} \qquad f(1)=-5$$

$$\lim_{h\to 0}\frac{\frac{1+h+4}{1+h-2}+5}{h}=\lim_{h\to 0}\frac{\frac{h+5}{h-1}+5}{h}=\lim_{h\to 0}\frac{\frac{h+5+5h-5}{h-1}}{h}=\lim_{h\to 0}\frac{\frac{6h}{h-1}}{h}=\lim_{h\to 0}\frac{6h}{h(h-1)}=\lim_{h\to 0}\frac{6}{h-1}=-6$$

In a piecewise function, if the derivative approaching from the right side of the value does not equal the derivative approaching from the left side of the function, the derivative does not exist.

Let's get ridiculous:

Find $y'\ with\ y=(3x-1)^4$ at $x=2$ using any of the above three methods.

Use the second method.

$$\lim_{x\to a}\frac{f(x)-f(a)}{x-a} \qquad f(2)=(3(2)-1)^4=625$$

$$\lim_{x\to 2}\frac{(3x-1)^4-625}{x-2}=\lim_{x\to 2}\frac{((3x-1)^2-25)((3x-1)^2+25)}{x-2}$$

$$\lim_{x\to 2}\frac{\big((3x-1)-5\big)((3x-1)+5)((3x-1)^2+25)}{x-2}$$

$$\lim_{x\to 2}\frac{((3x-6)(3x+4)((3x-1)^2+25)}{x-2}=\lim_{x\to 2}\frac{(3(x-2))(3x+4)((3x-1)^2+25)}{x-2}$$

$$\lim_{x\to 2}(3)(3x+4)((3x-1)^2+25)=3(10)(50)=1500$$