Calculus 3.1 – 3.5 Review

What does differentiable mean? You can take a derivative.

Differentiability implies continuity.

What are the four ways a function can be non-differentiable? Give graphs that demonstrate.

Corner $y = |x|$

Cusp $y = x^{\frac{3}{2}}$

Vertical Tangent $y = tan^{-1}x$

Discontinuity $y = \frac{x}{x}$

What is the Intermediate Value Theorem for Derivatives?

If $f'(3) = 10$ *and* $f'(12) = 20$, for every derivative between 10 & 20, there is an x value between 23 and 12 to create it.

What is the derivative of each of the following functions?

$y = 7$ $\quad y' = 0$

$y = 9x$ $\quad y' = 9$

$y = 5x^3 - 9x + 1$ $\quad y' = 15x^2 - 9$

$y = e^{\pi}$ $\quad y' = 0$

$y = \sqrt{x}$ $\quad y = x^{\frac{1}{2}}$ $\quad y' = \frac{1}{2}x^{-\frac{1}{2}}$

$y = \frac{1}{\sqrt[4]{x^7}}$ $\quad y = x^{-\frac{7}{4}}$ $\quad y' = -\frac{7}{4}x^{-\frac{11}{4}}$

$y = \frac{1}{x^5}$ $\quad y = x^{-5}$ $\quad y' = -5x^{-6}$

$y = (7x^2 + 8x - 2)(4x^2 + 9x - 1)$

$y' = (7x^2 + 8x - 2)(8x + 9) + (14x + 8)(4x^2 + 9x - 1)$

$y = (6x^3 + 8x)(\sqrt[3]{x}) = 6x^{\frac{10}{3}} + 8x^{\frac{4}{3}}$ $\quad y' = 20x^{\frac{7}{3}} + \frac{32}{3}x^{\frac{1}{3}}$

$y = \frac{2x-9}{8x+1}$ $y' = \frac{2(8x+1)-8(2x-9)}{(8x+1)^2} = \frac{16x+2-16x+72}{(8x+1)^2} = \frac{74}{(8x+1)^2}$

$y = \frac{2\sqrt{x}+7}{4\sqrt{x}-1}$

$top = 2\sqrt{x} + 7$ $bottom = 4\sqrt{x} - 1$

$top' = 2\left(\frac{1}{2\sqrt{x}}\right) = \frac{1}{\sqrt{x}}$ $bottom' = 4\left(\frac{1}{2\sqrt{x}}\right) = \frac{2}{\sqrt{x}}$

$$\frac{(4\sqrt{x}-1)\left(\frac{1}{\sqrt{x}}\right)-(2\sqrt{x}+7)\left(\frac{2}{\sqrt{x}}\right)}{(4\sqrt{x}-1)^2} = \frac{4\sqrt{x}-1-4\sqrt{x}-14}{\sqrt{x}(4\sqrt{x}-1)^2} = \frac{-15}{\sqrt{x}(4\sqrt{x}-1)^2}$$

Use $\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$ to find $f'(2)$ with $f(x) = \sqrt{x+2}$.

$$\lim_{h\to 0}\frac{\sqrt{x+h+2}-\sqrt{x+2}}{h}\cdot\frac{\sqrt{x+h+2}+\sqrt{x+2}}{\sqrt{x+h+2}+\sqrt{x+2}} = lim_{h\to 0}\frac{x+h+2-x-2}{h(\sqrt{x+h+2}+\sqrt{x+2})} =$$

$$lim_{h\to 0}\frac{h}{h(\sqrt{x+h+2}+\sqrt{x+2})} = lim_{h\to 0}\frac{1}{\sqrt{x+h+2}+\sqrt{x+2}} = \frac{1}{2\sqrt{x+2}}$$

$$\frac{1}{2\sqrt{2+2}} = \frac{1}{2(2)} = \frac{1}{4}$$

Use $\lim_{h\to 0}\frac{f(a+h)-f(a)}{h}$ to find $f'(3) = \frac{1}{x+6}$.

$$\lim_{h\to 0}\frac{\frac{1}{3+h+6}-\frac{1}{9}}{h} = \lim_{h\to 0}\frac{\frac{1}{9+h}-\frac{1}{9}}{h} = \lim_{h\to 0}\frac{\frac{9-9-h}{9(9+h)}}{h} = \lim_{h\to 0}\frac{\frac{-h}{9(9+h)}}{h} = \lim_{h\to 0}\frac{-h}{9h(9+h)} =$$

$$\lim_{h\to 0} -\frac{1}{9(9+h)} = -\frac{1}{81}$$

If $x(t) = x^3 - 15x^2 + 72x + 2$ is the position function for a particle, find the following:

Velocity: Find the derivative of the position function. $v(t) = 3x^2 - 30x + 72$

Acceleration: Find the derivative of the velocity function. $a(t) = 6x - 30$

When does the particle change direction? Set the velocity = 0.

$v(t) = 3x^2 - 30x + 72 = 0 \quad x^2 - 10x + 24 = 0 \quad (x-6)(x-4) = 0$

$x = 4, 6$

What is the displacement over the first 10 seconds?

Final position – initial position.

$x(10) = 222 \quad x(0) = 2 \qquad x(10) - x(2) = 220$

What is the total distance travelled over the first 10 seconds?

$x(4) - x(0) = 114 - 2 = 112$

$x(6) - x(4) = 110 - 114 = -4 = 4$

$x(10) - x(6) = 222 - 110 = 112$

Total distance equals $112 + 4 + 112 = 228$

What is the velocity when the acceleration is 0?

Let the acceleration = 0, solve for t and put that t value into the velocity function.

$a(t) = 6t - 30 = 0 \quad 6t = 30 \quad t = 5$

$v(5) = -3$

What direction is the particle going when $t = 3$?

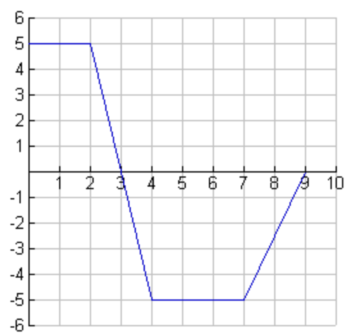
Look at the velocity function at t = 3.

If it is positive, the particle is moving right.

If it is negative, the particle is moving left.

$v(3) = 9$ The particle is moving to the right.

This is a velocity graph.



At $t = 1$,

The particle is traveling at what velocity? 5

The particle is traveling at what speed? 5

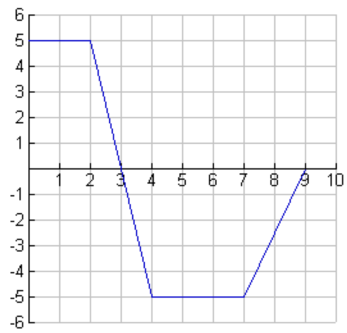
The particle is traveling in what direction? right

What is the acceleration? 0

Is the particle right or left of its original position? right

The particle is slowing down, speeding up or staying at a constant speed?

Staying at a constant speed.



At $t = 6$,

The particle is traveling at what velocity? -5

The particle is traveling at what speed? 5

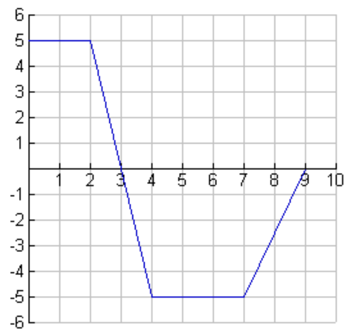
The particle is traveling in what direction? left

What is the acceleration? 0

Is the particle right or left of its original position? At the original position.

The particle is slowing down, speeding up or staying at a constant speed?

Staying at a constant speed.



At $t = 8$,

The particle is traveling at what velocity? -2.5

The particle is traveling at what speed? 2.5

The particle is traveling in what direction? left

What is the acceleration? 2.5

Is the particle right or left of its original position? left

The particle is slowing down, speeding up or staying at a constant speed?

The particle is slowing down.

If $v(t) = 9$ & $a(t) = -4$, the particle is slowing down to the right.

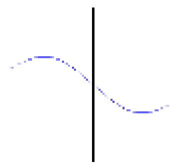
If $v(t) = -9$ & $a(t) = -4$, the particle is Speeding up to the left.

If $v(t) = -9$ & $a(t) = 4$, the particle is Slowing down to the left.

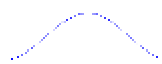
What is the derivative of $y = sinx$ and prove it.

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\sin(x+h) - sinx}{h} = \lim_{h \to 0} \frac{sinxcosh + cosxsinh - sinx}{h}$$

$$\lim_{h \to 0} \frac{sinxcosh - sinx}{h} + \lim_{h \to 0} \frac{cosxsinh}{h} = sinx \lim_{h \to 0} \frac{cosh - 1}{h} + cosx \lim_{h \to 0} \frac{sinx}{x}$$



$\lim_{h \to 0} \frac{cosh-1}{h} = 0$ because of this graph.



$\lim_{h \to 0} \frac{sinx}{x} = 1$ because of this graph

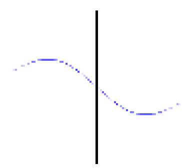
$sinx(0) + cosx(1) = cosx$

What is the derivative of $y = cosx$ and prove it.

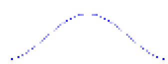
$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{cosxcosh - sinxsinh - cosx}{h}$$
$$= \lim_{h\to 0}\frac{cosxcosh - cosx}{h} - \lim_{h\to 0}\frac{sinxsinh}{h}$$

$$\lim_{h\to 0}\frac{cosx(cosh-1)}{h} - \lim_{h\to 0}\frac{sinxsinh}{h} =$$

$$cosx\lim_{h\to 0}\frac{(cosh-1)}{h} - sinx\lim_{h\to 0}\frac{sinh}{h} =$$



$\lim_{h\to 0}\frac{(cosh-1)}{h} = 0$ because of the graph



$\lim_{h\to 0}\frac{sinh}{h} = 1$ because of the graph

$cosx(0) - sinx(1) = -sinx$

What is the derivative of $y = tanx$ and prove it.

$y = \frac{sinx}{cosx}$

Use the quotient rule to find the derivative.

$$\frac{BDT - TDB}{B^2} = \frac{cosx(cosx) - sinx(-sinx)}{cos^2x} = \frac{cos^2x + sin^2x}{cos^2x} = \frac{1}{cos^2x} = sec^2x$$

Find the derivative of $y = cotx$ and prove it.

$y = \frac{cosx}{sinx}$

Use the quotient rule to find the derivative.

$$\frac{BDT - TDB}{B^2} = \frac{sinx(-sinx) - cosx(-cosx)}{cos^2x} = \frac{-sin^2x - cos^2x}{sin^2x} =$$

$$\frac{-(sin^2x + cos^2x)}{sin^2x} = \frac{-1}{sin^2x} = -csc^2x$$

Find the derivative of $y = secx$ and prove it.

$y = \frac{1}{cosx}$

Use the quotient rule to find the derivative.

$$\frac{BDT - TDB}{B^2} = \frac{cosx(0) - 1(-sinx)}{cos^2x} = \frac{sinx}{cos^2x} = \frac{sinx}{cosx} \cdot \frac{1}{cosx} = tanxsecx = secxtanx$$

Find the derivative of $y = cscx$ and prove it.

$y = \frac{1}{sinx}$

Use the quotient rule to find the derivative.

$$\frac{BDT - TDB}{B^2} = \frac{sinx(0) - 1(cosx)}{sin^2x} = \frac{-cosx}{sin^2x} = \frac{-cosx}{sinx} \cdot \frac{1}{sinx} = -cotxcscx = -cscxcot$$

Function	Derivative
$y = sinx$	$y' = cosx$
$y = cosx$	$y' = -sinx$
$y = tanx$	$y' = sec^2x$
$y = cotx$	$y' = -csc^2x$
$y = secx$	$y' = secxtanx$
$y = cscx$	$y' = -cscxcotx$
$y = x^n$	$y' = nx^{n-1}$
$y = f(x) + g(x)$	$y' = f'(x) + g'(x)$
$y = (\sqrt{7})^3$	$y' = 0$
$y = f(x)g(x)$	$y' = f'(x)g(x) + f(x)g'(x)$
$y = \frac{f(x)}{g(x)}$	$y' = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$
$y = f(g(h(x)))$	$y' = f'\left(g\left(h(x)\right)\right)g'\left(h(x)\right)h'(x)$