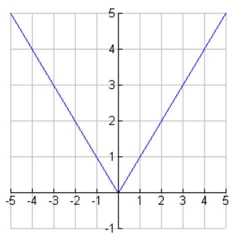
Calculus 3.1 – 3.3 Review

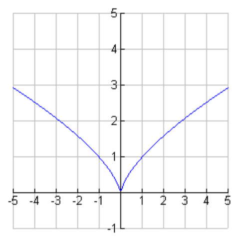
What does differentiable mean? You can take a derivative.

Differentiability implies continuity.

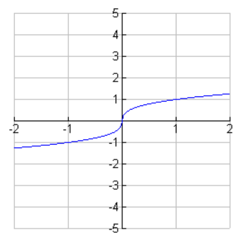
What are the four ways a function can be non-differentiable? Give graphs that demonstrate.



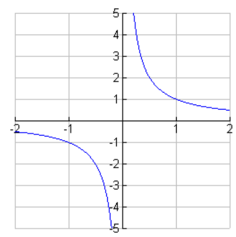
corner



cusp



vertical tangent



discontinuity

What is the Intermediate Value Theorem for Derivatives?

If $f'(0) = -5$ and $f'(10) = 13$,

For every derivative between -5 & 13, there is a $f'(c)$ with $0 \le c \le 10$ to create it.

What is the derivative of each of the following functions?

$y = 7$ $\quad$ $y' = 0$

$y = 9x$ $\quad$ $y' = 9$

$y = 5x^3 - 9x + 1$ $\quad$ $y' = 15x^2 - 9$

$y = e^{\pi}$ $\quad$ $y' = 0$

$y = \sqrt{x}$ $\quad$ $y = x^{\frac{1}{2}}$ $\quad$ $y' = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

$y = \frac{1}{\sqrt[4]{x^7}}$ $\quad$ $y = x^{-\frac{7}{4}}$ $\quad$ $y' = -\frac{7}{4}x^{-\frac{11}{4}} = -\frac{7}{4x^{\frac{11}{4}}}$

$y = \frac{1}{x^5}$ $\quad$ $y = x^{-5}$ $\quad$ $y' = -5x^{-6} = -\frac{5}{x^6}$

$y = (7x^2 + 8x - 2)(4x^2 + 9x - 1)$ $\quad$ $7x^2 + 8x - 2$ $\quad$ $4x^2 + 9x - 1$

$14x + 8$ $\quad$ $8x + 9$

$$(7x^2 + 8x - 2)(8x + 9) + (4x^2 + 9x - 1)(14x + 8)$$

$$56x^3 + 63x^2 + 64x^2 + 72x - 16x - 18 + 56x^3 + 32x^2 + 126x^2 + 72x - 14x - 8$$

$$y' = 112x^3 + 285x^2 + 114x - 26$$

$y = (6x^3 + 8x)(\sqrt[3]{x})$ $\quad$ $6x^3 + 8x$ $\quad$ $\sqrt[3]{x}$

$18x^2 + 8$ $\quad$ $(x)^{\frac{1}{3}}$

$\frac{1}{3}x^{-\frac{2}{3}}$

$\frac{1}{3x^{\frac{2}{3}}}$

$$\left(\frac{6x^3+8x}{3x^{\frac{2}{3}}}\right) + \frac{(18x^2+8)(\sqrt[3]{x})}{1}$$

$$\frac{6x^3+8x+\left(3x^{\frac{2}{3}}\right)((18x^2+8)(\sqrt[3]{x})}{3x^{\frac{2}{3}}}$$

$$\frac{6x^3+8x+(3x)(18x^2+8)}{3x^{\frac{2}{3}}} = \frac{6x^3+8x+(54x^3+24x)}{3x^{\frac{2}{3}}}$$

$$y' = \frac{60x^3+32x}{3x^{\frac{2}{3}}}$$

$y = \frac{2x-9}{8x+1}$

$top = 2x - 9 \qquad bot = 8x + 1$

$top' = 2 \qquad bot' = 8$

$\frac{2(8x+1)-(2x-9)8}{(8x+1)^2} = \frac{16x+2-16x+72}{(8x+1)^2} = \frac{74}{(8x+1)^2}$

$y' = \frac{74}{(8x+1)^2}$

$y = \frac{2\sqrt{x}+7}{4\sqrt{x}-1}$

$top = 2\sqrt{x} + 7 \qquad bot = 4\sqrt{x} - 1$

$top' = \frac{1}{\sqrt{x}} \qquad bot' = \frac{2}{\sqrt{x}}$

$\frac{\frac{4\sqrt{x}-1}{\sqrt{x}} - \frac{2(2\sqrt{x}+7)}{\sqrt{x}}}{(4\sqrt{x}-1)^2} = \frac{\frac{4\sqrt{x}-1-4\sqrt{x}-14}{\sqrt{x}}}{(4\sqrt{x}-1)^2} = \frac{-15}{\sqrt{x}(4\sqrt{x}-1)^2} =$

Use $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ to find $f'(2)$ with $f(x) = \sqrt{x+2}$.

$\lim_{h\to 0} \frac{\sqrt{x+h+2}-\sqrt{x+2}}{h} \cdot \frac{\sqrt{x+h+2}+\sqrt{x+2}}{\sqrt{x+h+2}+\sqrt{x+2}} = \lim\limits_{h\to 0} \frac{x+h+2-x-2}{h(\sqrt{x+h+2}+\sqrt{x+2})} =$

$\lim\limits_{h\to 0} \frac{h}{h(\sqrt{x+h+2}+\sqrt{x+2})} = \lim\limits_{h\to 0} \frac{1}{(\sqrt{x+h+2}+\sqrt{x+2})} = \frac{1}{2\sqrt{x+2}}$ at $x = 2 \quad = \frac{1}{4}$

Use $\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ to find $f'(3)$ with $f(x) = \frac{1}{x+6}$.

$\lim_{h\to 0} \frac{\frac{1}{3+h+6} - \frac{1}{9}}{h} = \lim\limits_{h\to 0} \frac{\frac{9-9-h}{9(9+h)}}{h} = \lim\limits_{h\to 0} -\frac{h}{9h(9+h)} = -\frac{1}{9(9)} = -\frac{1}{81}$

Use $\lim_{x\to 1}\frac{f(x)-f(a)}{x-a}$ to find $f'(1)$ if $f(x) = (6x-5)^4 - 81$

$$\lim_{x\to 1}\frac{(6x-5)^4-81-(-80)}{x-1} = \lim_{a\to 1}\frac{(6x-5)^4-1}{x-1} = \lim_{a\to 1}\frac{((6x-5)^2+1)((6x-5)^2-1)}{x-1}$$

$$\lim_{x\to 1}\frac{((6x-5)^2+1)((6x-5))-1)((6x-5)+1)}{x-1} = \lim_{x\to 1}\frac{((6x-5)^2+1)(6x-6)(6x-4)}{x-1} =$$

$$\lim_{x\to 1}\frac{((6x-5)^2+1)6(x-1)(6x-4)}{x-1} = \lim_{x\to 1}(6x-5)^2+1)6(6x-4) = 24$$