2.4 Notes

Weight Loss Chart

The average rate of change is the slope between two points on a curve.

The average rate of lost weight would be computed using which two days?

May 10th and September 1st

Does this describe my weekly loss?

A weekly or daily rate of loss would be a better indicator.

How can you find the slope of a line at one point?

$$lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Use a graph to create two points, $(x, f(x))$ & $\left(x+h, f(x+h)\right)$, and find the slope. As $h \to 0$, this becomes one point.

Find the slope of $f(x) = x^2$ using the above definition.

$$lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = lim_{h\to 0}\frac{(x+h)^2-x^2}{h} = lim_{h\to 0}\frac{(x^2+2xh+h^2)-x^2}{h}$$

$$lim_{h\to 0}\frac{2xh+h^2}{h} = lim_{h\to 0}(2x+h) = 2x$$

Find the slope at $x = 2, 4, 10$.

$2(2) = 4 \qquad 2(4) = 8 \qquad 2(10) = 20$

Find the slope at $x = 0$.

$2(0) = 0$

Find the slopes at $x = -2, -4\ and -10$.

$2(-2) = -4 \quad 2(-4) = -8 \quad 2(-10) = -20$

Graph the function $y = x^2$ and show the tangent lines.

The slopes going from left to right are negative, then 0, then positive.

What does this make the vertex of the parabola?

Create a graph where the slopes go from negative to 0 to positive.

What is the point where the slope is 0?

The average rate of change is the slope between two points on a curve.

Use the formula $\frac{y_2 - y_1}{x_2 - x_1}$.

The slope at one point or instantaneous rate of change is

$$lim_{h \to 0} \frac{f(x+h) - f(x)}{h}.$$

Tangent lines to curves

Given a point and the equation of the curve, find the slope at the point and then create an equation using the point slope formula.

Find the equation of the tangent line at $x = 4\ for\ y = x^3$.

$$lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = lim_{h\to 0}\frac{(x+h)^3-x^3}{h} = lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3-x^3}{h}$$

$$lim_{h\to 0}\frac{3x^2h+3xh^2+h^3}{h} = lim_{h\to 0}\frac{h(3x^2+3xh+h^2)}{h} =$$

$$lim_{h\to 0}(3x^2 + 3xh + h^2) = 3x^2$$

At $x = 4$, the slope is $3(4)^2 = 48$.

At $x = 4, y = 4^3 = 64$.

The equation of the tangent line is $y - 64 = 48(x - 4)$.

Normal lines to curves

Given a point and the equation of the curve, find the slope at the point, find the perpendicular slope (take the negative reciprocal of the slope) and then create an equation using the point slope formula.

Find the equation of the normal line at $x = 3\ for\ y = \frac{1}{x}$.

$$lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = lim_{h\to 0}\frac{\frac{1}{x+h}-\frac{1}{x}}{h} = lim_{h\to 0}\frac{\frac{x-(x+h)}{x(x+h)}}{h} = lim_{h\to 0}\frac{\frac{-h}{x(x+h)}}{h}$$

$$lim_{h\to 0}\frac{-h}{xh(x+h)} = lim_{h\to 0}\frac{-1}{x(x+h)} = \frac{-1}{x(x)} = -\frac{1}{x^2}$$

At $x = 3$ the slope is $-\frac{1}{9}$. The perpendicular slope is 9. At $x = 3, y = \frac{1}{3}$.

The equation of the normal line is $y - \frac{1}{3} = 9(x - 3)$.