2.3 Continuity

Explain continuity to a third grader.

Draw a graph and do not pick up your pencil.

The three mathematical conditions for continuity:

1) For every value of c in the function, $f(c)$ exists.
2) For value of c, $lim_{x\to c} f(x)$ exists.
3) $lim_{x\to c} f(x) = f(c)$

The three mathematical conditions in high schoolian:

1) Every x has a y value
2) Every x has a limit
3) Every limit of an x equals the value of the x.

Draw a graph and give an equation of a function for each type of discontinuity.

1) Jump

   Piecewise or $f(x) = [x]$

2) Infinite

   $f(x) = \frac{1}{x}$ $\quad$ $f(x) = \frac{x-2}{x+3}$

3) Oscillating

   $f(x) = \sin\left(\frac{1}{x}\right)$

4) Removable

   $f(x) = \frac{x-2}{x-2}$

If two functions are continuous, their sum is continuous.

All composites of continuous functions are continuous.

If $f(x)$& $g(x)$ are continuous, $f\big(g(x)\big)$& $g(f(x))$ are continuous.

A continuous function is continuous at every point in its domain.

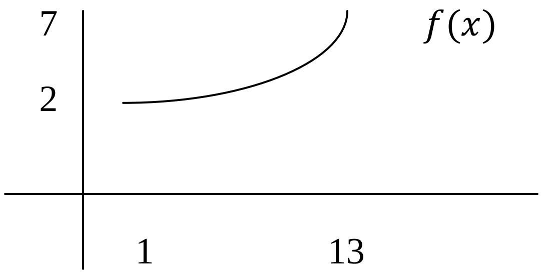
Is $f(x) = \frac{1}{x}$ a continuous function?

Yes. The function is discontinuous at $x = 0$ but 0 is not in the domain.

Is $f(x) = [x]$ a continuous function?

No. This is a classic jump function. The jumps are within the domain.

Intermediate Value Theorem



If $f(x)$ is continuous over [1, 13], for any y value between [2, 7], there must be an x value between 1 & 13 to create it.

The mathy version:

If $f(x)$ is continuous over $[a, b]$, for any y_0 such that $f(a) \leq y_0 \leq f(b)$, there must be a c such that $a \leq c \leq b$, such that $f(c) = y_0$.