2.2 Limits Involving Infinity

Bill Gates would look at the person with the most power in the room.

When you are looking at a limit approaching infinity with a rational expression, find the highest power in the numerator and the highest power in the denominator. Take the limit of this fraction as x approaches infinity.

$$lim_{x\to\infty}\frac{x^3+x^2+x+4}{x^2+3x+1}\rightarrow\ lim_{x\to\infty}\frac{x^3}{x^2}=lim_{x\to\infty}x=\infty$$

$$lim_{x\to\infty}\frac{5x^2+7x+2}{4x^2+3x+1}\rightarrow\ lim_{x\to\infty}\frac{5x^2}{4x^2}=lim_{x\to\infty}\frac{5}{4}=\frac{5}{4}$$

$$lim_{x\to\infty}\frac{3x^2+4x+8}{2x^3+x^2-5x+13}\rightarrow\ lim_{x\to\infty}\frac{3x^2}{2x^3}=lim_{x\to\infty}\frac{3}{2x}=\frac{3}{huge}=0$$

Quick Bill Gates Version:

$$lim_{x\to\infty}\frac{ax^b}{cx^d}$$

$$b>d\ \rightarrow\ \infty$$

$$b=d\ \rightarrow\ \frac{a}{c}$$

$$b<d\ \rightarrow\ 0$$

Mathematical Version of Bill Gates:

Divide each term by the highest power of x. Find the limit as x approaches infinity.

$$lim_{x\to\infty}\frac{5x^2+7x+2}{4x^2+3x+1}=lim_{x\to\infty}\frac{\frac{5x^2}{x^2}+\frac{7x}{x^2}+\frac{2}{x^2}}{\frac{4x^2}{x^2}+\frac{3x}{x^2}+\frac{1}{x^2}}=lim_{x\to\infty}\frac{5+\frac{7}{x}+\frac{2}{x^2}}{4+\frac{3}{x}+\frac{1}{x^2}}=$$

$$lim_{x\to\infty}\frac{5+0+0}{4+0+0}=\frac{5}{4}$$

The best way to find a horizontal asymptote is to use Bill Gates.

If the limit equals infinity, there is no horizontal asymptote.

If the limit equals a value, the asymptote is $y = value$.

End Behavior Models

You are trying to make a complicated function approximate to a simpler function as x approaches infinity.

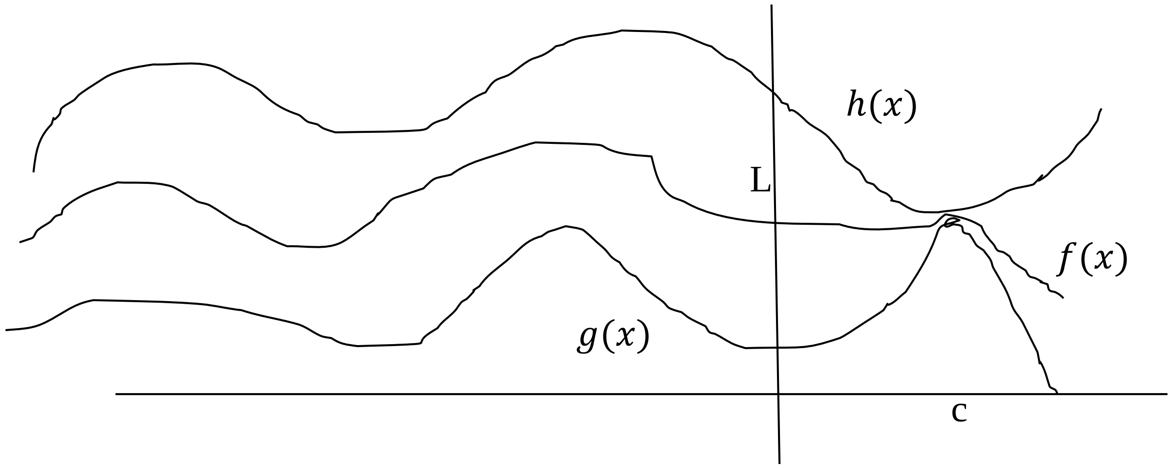
The function $f(x) = x^3 + 7x^2 - 9x + 1$ will approach the function $g(x) = x^3$ as x approaches positive infinity.

The right end behavior model for $f(x)$ is $g(x)$.

If $lim_{x \to \infty} \frac{f(x)}{g(x)} = 1$ then $g(x)$ is the right end behavior model for $f(x)$.

If $lim_{x \to -\infty} \frac{f(x)}{h(x)} = 1$ then $h(x)$ is the left end behavior model for $f(x)$.

The Sandwich Theorem



A peanut butter sandwich (no jelly) has two pieces of bread and peanut butter in between.

The bread is the functions $h(x)$& $g(x)$.

The peanut butter is the function $f(x)$.

If $h(x) \geq f(x) \geq g(x)$ and

$lim_{x \to c} h(x) = L$ and $lim_{x \to c} g(x) = L$, then $lim_{x \to c} f(x) = L$

Example

$lim_{x \to \infty} \frac{sinx}{x} =$

$-1 \leq sinx \leq 1$ Dividing by x creates $-\frac{1}{x} \leq \frac{sinx}{x} \leq \frac{1}{x}$

The bread are the functions $-\frac{1}{x}$ & $\frac{1}{x}$.

The peanut butter is the function $\frac{sinx}{x}$.

If $lim_{x \to \infty} -\frac{1}{x} = 0$ & $lim_{x \to \infty} \frac{1}{x} = 0$, then $lim_{x \to \infty} \frac{sinx}{x} = 0$.

Another Example

$$lim_{x\to\infty}\frac{5x+sinx}{x}$$

Use divide and conquer. This means create two fractions.

$$lim_{x\to\infty}\frac{5x}{x}+lim_{x\to\infty}\frac{sinx}{x}=lim_{x\to\infty}5+lim_{x\to\infty}\frac{sinx}{x}=5+0=5$$