2.1 Limits

What does $lim_{x \to c} f(x) = L$ mean?

As x approaches c, the $f(x)$ or y value approaches L.

$lim_{x \to 4} x^2 = 16$

As x approaches 4, $x^2 (4^2)$ approaches 16.

Approaches means from both sides of 4.

From the left side 3, 3.5, 3.75, 3.9, 3.95, 3.999 → 4

From the right side 5, 4.5, 4.25, 4.1, 4.09, 4.05, 4.001 → 4

Properties of limits

If $lim_{x \to c} f(x) = L, lim_{x \to c} g(x) = M$

You can add them $lim_{x \to c} f(x) + lim_{x \to c} g(x) = L + M$

You can subtract them $lim_{x \to c} f(x) - lim_{x \to c} g(x) = L - M$

You can multiply them $lim_{x \to c} (f(x))(g(x)) = L \cdot M$

You can divide them $lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{M}$

You can factor out a constant $lim_{x \to c} kf(x) = kL$

You can raise them to a power $lim_{x \to c} (f(x))^p = L^p$

There are three ways to find a limit as x approaches a value.

1) $lim_{x\to 3}(x^2 + 9x - 2)$ Just plug it in.
   $3^2 + 9(3) - 2 = 34$

2) $lim_{x\to 0}\frac{x}{x^2+3x}$
   Do algebra (usually factoring), cancel something out and then just plug it in.
   $$lim_{x\to 0}\frac{x}{x(x+3)} = lim_{x\to 0}\frac{1}{(x+3)} = \frac{1}{3}$$
   $$lim_{x\to 2}\frac{x^2-4x+4}{x^2-3x+2} = lim_{x\to 2}\frac{(x-2)(x-2)}{(x-2)(x-1)} = lim_{x\to 2}\frac{x-2}{2-1} = 0$$
   $$lim_{x\to 4}\frac{x-4}{\sqrt{x}-2} = lim_{x\to 4}\frac{(\sqrt{x}-2)(\sqrt{x}+2)}{\sqrt{x}-2} = lim_{x\to 4}\sqrt{x}+2 = 4$$

3) $lim_{x\to 0}\frac{sinx}{x}$ Graph it or memorize it.
   $lim_{x\to 0}\frac{sinx}{x} = 1$ $lim_{x\to 0}\frac{cosx-1}{x} = 0$

One Sided Limits

What does this mean? $lim_{x \to c^-} f(x)$

What does the y value approach as the x value approaches c from the left (negative) side?

This is called the left-side limit.

What does this mean? $lim_{x \to c^+} f(x)$

What does the y value approach as the x value approaches c from the right (positive) side?

This is called the right-side limit.

Create a function where the left-side limit does not equal the right-side limit.

$f(x) = \frac{1}{x}$

$lim_{x \to 0^-} \frac{1}{x} = -\infty$ $lim_{x \to 0^+} \frac{1}{x} = \infty$

If the left-side limit does not equal the right-side limit, the limit does not exist.

The mathy version of this:

If $lim_{x \to c^-} f(x) \neq lim_{x \to c^+} f(x),$ then $lim_{x \to c} f(x)$ does not exist.

What is $lim_{x \to 2} f(x)$? $f(x) = \begin{cases} x^2 - 3x + 1 & if\ x < 2 \\ -|x - 3| & if\ x \geq 2 \end{cases}$

$x < 2$ is the left side limit

$x > 2$ is the right side limit

We have to see if both of these are the same. Plug in the 2 to both functions and you get -1 for both. Since the left-side limit equals the right-side limit, the limit exists.

Create a graph where the y-value of a function equals the limit of the function.

Create a graph where the y-value of a function exists but does it not equal the limit of the function. The limit exists.

Create a graph where the y-value of a function does not exist but the limit exists at that x-value.