Calculus 2.1 – 2.4 Notes

Find the limit showing the work.

$\lim_{x\to 2}(6x-4) = 6(2) - 4 = 8$

$\lim_{x\to 2}\frac{x-2}{x^2-4} = \lim_{x\to 2}\frac{x-2}{(x+2)(x-2)} = \lim_{x\to 2}\frac{1}{x+2} = \frac{1}{4}$

$\lim_{x\to 4}\frac{x-4}{\sqrt{x}-2} = \lim_{x\to 4}\frac{(\sqrt{x}-2)(\sqrt{x}+2)}{\sqrt{x}-2} = \lim_{x\to 4}\sqrt{x}+2 = 4$

$\lim_{x\to 0}\frac{sin5x}{x} = \lim_{x\to 0}\frac{5sin5x}{5x} = 5\lim_{x\to 0}\frac{sin5x}{5x} = 5(1) = 5$

$\lim_{x\to\infty}\frac{x}{x+17}$ Bill Gates → Powers are equal, do coefficients 1

$\lim_{x\to\infty}\frac{x^3}{x^2+9x-17}$ Bill Gates → Bigger power on top ∞

$\lim_{x\to\infty}\frac{x^2+10{,}000}{x^4+x^3+17}$ Bill Gates → Bigger power on bottom 0

Explain the three different possible answers for Bill Gates.

$\lim_{x\to\infty}\frac{ax^b}{cx^d} =$

If b > d, the limit goes to ∞

If d > b, the limit goes to 0

If d = b, the limit goes to $\frac{a}{c}$

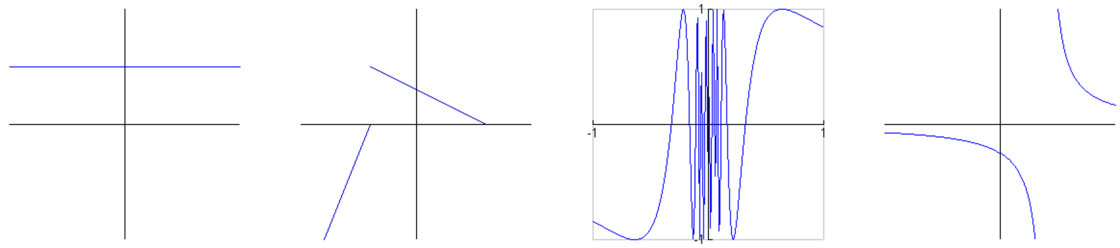
What are the vertical and horizontal asymptotes for $f(x) = \frac{x^3-2x^2+x}{x^2-x}$?

$\frac{x^3-2x^2+x}{x^2-x} = \frac{x(x^2-2x+1)}{x(x-1)} = \frac{x(x-1)(x-1)}{x(x-1)} = x-1$

There are no vertical asymptotes only holes at x = 1.

There is no horizontal(use Bill gates).

What are the four types of discontinuities? Use graphs to describe them.



$y = \frac{x-2}{x-2}$		$y = \sin(\frac{1}{x})$	$y = \frac{1}{x-2}$
Removeable	Jump	Oscillating	Infinite

What is the easiest way to see if a graph is continuous?

If the graph is continuous, you will not have to pick up your pencil.

What are the three mathematical conditions to show that a function is continuous?

$f(a) exists, \lim_{x \to a} f(x) exists, \ f(a) = \lim_{x \to a} f(x)$

Is $f(x) = \frac{x+4}{x(x-4)}$ a continuous function? Explain why or why not.

Yes. It is continuous within its domain.

Create a function that has a removable discontinuity and then remove the discontinuity.

$y = \frac{x-5}{x-5}$ $\qquad$ $y = \begin{cases} \frac{x-5}{x-5} & if \ \ x \neq 5 \\ 1 & if \ x = 5 \end{cases}$ $\qquad$ OR $\qquad$ $y = 1$

If $\lim_{x \to 2^+} f(x) \neq \lim_{x \to 2^-} f(x) \ then \ \lim_{x \to 2} f(x)$ does not exist.

Draw a graph with the following conditions.

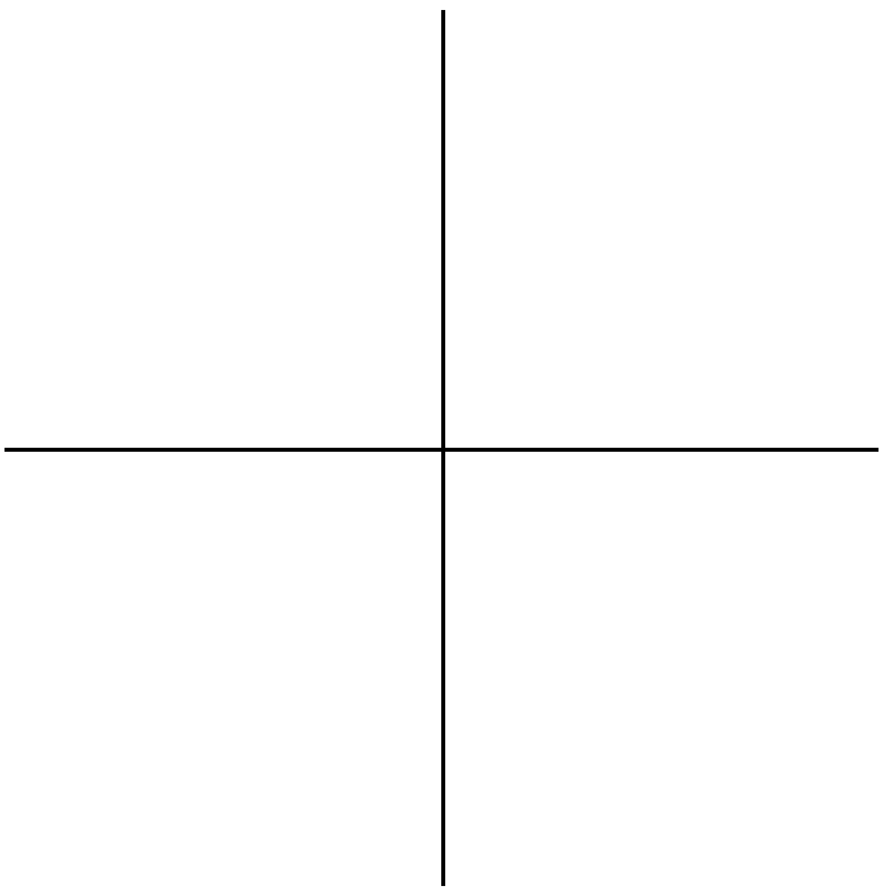
$\lim_{x\to 2^+} f(x) = 4$ $\lim_{x\to 2^-} f(x) = -5$ $f(2) = 0$

$\lim_{x\to -5^+} f(x) = \infty$ $\lim_{x\to -5^-} f(x) = \infty$

$\lim_{x\to 8^+} f(x) = \infty$ $\lim_{x\to 8^-} f(x) = -\infty$

$\lim_{x\to -1} f(x) = -3$ $f(-1) = 7$

$\lim_{x\to \infty} f(x) = 0$ $\lim_{x\to -\infty} f(x) = 4$



Find the derivative of the following functions using the definition of the derivative.

$f(x) = x^2 - 7x + 5$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{(x+h)^2-7(x+h)+5-(x^2-7x+5)}{h}$$

$$\lim_{h\to 0} \frac{x^2+2xh+h^2-7x-7h+5-x^2+7x-5}{h} = \lim_{h\to 0} \frac{2xh+h^2-7h}{h} = \lim_{h\to 0}(2x+h-7) = 2x-7$$

$f(x) = \dfrac{1}{x+9}$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\frac{1}{x+9+h}-\frac{1}{x+9}}{h} = \lim_{h\to 0} \frac{\frac{x+9-x-9-h}{(x+9+h)(x+9)}}{h} = \lim_{h\to 0} \frac{\frac{-h}{(x+9+h)(x+9)}}{h}$$

$$\lim_{h\to 0} \frac{-h}{h(x+9+h)(x+9)} = \lim_{h\to 0} \frac{-1}{(x+9+h)(x+9)} = -\frac{1}{(x+9)^2}$$

$f(x) = \sqrt{x-6}$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sqrt{x+h-6}-\sqrt{x-6}}{h} \cdot \frac{\sqrt{x+h-6}+\sqrt{x-6}}{\sqrt{x+h-6}+\sqrt{x-6}} = \lim_{h\to 0} \frac{x+h-6-x+6}{h(\sqrt{x+h-6}+\sqrt{x-6})}$$

$$\lim_{h\to 0} \frac{h}{h(\sqrt{x+h-6}+\sqrt{x-6})} = \lim_{h\to 0} \frac{1}{\sqrt{x+h-6}+\sqrt{x-6}} = \frac{1}{2\sqrt{x-6}}$$

$f(x) = sinx$

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{\sin(x+h)-sinx}{h} = \lim_{h\to 0} \frac{sinxcosh+cosxsinh-sinx}{h}$$

$$\lim_{h\to 0} \frac{sinxcosh-sinx}{h} + \lim_{h\to 0} \frac{cosxsinh}{h} = sinx\lim_{h\to 0} \frac{cosh-1}{h} + cosx\lim_{h\to 0} \frac{sinh}{h}$$

$$sinx(0) + cosx(1) = cosx$$

Find the equation of the tangent and normal lines to the curve $y = x^3 + 4$ at $x = 2$.

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h} = \lim_{h\to 0}\frac{(x+h)^3+4-x^3-4}{h}$$

$$\lim_{h\to 0}\frac{x^3+3x^2h+3xh^2+h^3+4-x^3-4}{h} = \lim_{h\to 0}\frac{3x^2h+3xh^2+h^3}{h}$$

$$\lim_{h\to 0}(3x^2+3xh+h^2) = 3x^2$$

When $x = 2, m = 3(2)^2 = 12, y = x^3 + 4 = 2^3 + 4 = 12$

$(2, 12)$ $m = 12$ $y - 12 = 12(x - 2)$ is the tangent line

$y - 12 = -\frac{1}{12}(x - 2)$ is the normal line