Review for 2.1 – 2.2 Limits

1) What does $lim_{x \to c} f(x) = L$ mean?
   As the x value approaches c, the y value approaches L.

2) If $lim_{x \to 3^-} f(x) = 8$ & $lim_{x \to 3^+} f(x) = 10$,
   The limit does not exist.

3) What are the three ways to find a limit?
   - Plug it in.
   - Algebra (factor)
   - Graph it.

4) $lim_{x \to 3} \frac{x^2 - 3x + 4}{2x - 7} =$

$$\frac{3^2 - 3(3) + 4}{2(3) - 7} = \frac{4}{-1} = -4$$

5) $lim_{x \to 3} \frac{x^2 - 9}{x - 3} =$

$$\frac{(x-3)(x+3)}{x-3} = lim_{x \to 3}(x+3) = 6$$

6) $lim_{x \to 3} \frac{x - 3}{\sqrt{x-3}} =$

$$lim_{x \to 3} \frac{x-3}{\sqrt{x-3}} \cdot \frac{\sqrt{x-3}}{\sqrt{x-3}} = lim_{x \to 3} \frac{(x-3)\sqrt{x-3}}{(x-3)}$$

$$= lim_{x \to 3} \sqrt{x-3} = 0$$

7) $lim_{x \to 9} \frac{\sqrt{x}-3}{x-9} \cdot \frac{\sqrt{x}+3}{\sqrt{x}+3} = lim_{x \to 9} \frac{x-9}{(x-9)(\sqrt{x}+3)} = lim_{x \to 9} \frac{1}{\sqrt{x}+3} = \frac{1}{6}$

8) $lim_{x\to0}\frac{sinx}{x}$ =1 from the graph

9) $lim_{x\to0}\frac{sin5x}{sin7x} = lim_{x\to0}\frac{sin5x}{5x}\cdot\frac{7x}{sin7x}\cdot\frac{5}{7} = 1\cdot1\cdot\frac{5}{7} = \frac{5}{7}$

10) $lim_{x\to0}\frac{sin^2x}{xcosx} = lim_{x\to0}\frac{sinx}{x}\cdot\frac{sinx}{cosx} = lim_{x\to0}(1)(tanx) = \tan(0) = 0$

11) $lim_{x\to0}\frac{1}{xcscx} = lim_{x\to0}\frac{sinx}{x} = 1$

12) $lim_{x\to0}\frac{sinx}{x^2-7x} = lim_{x\to0}\frac{sinx}{x(x-7)} = lim_{x\to0}\frac{sinx}{x}\cdot\frac{1}{x-7} = (1)\left(-\frac{1}{7}\right) = -\frac{1}{7}$

13) $lim_{x\to\infty}\frac{x^2+9x-2}{x+10{,}000} =$ Bill Gates Biggest power on the top means ∞.

14) $lim_{x\to\infty}\frac{2x}{5x^7-13} =$ Bill Gates Biggest power on the bottom means 0.

15) $lim_{x\to\infty}\frac{x^4+9x-1}{5x^4+3} =$ Billy $\frac{x^4}{5x^4} = \frac{1}{5}$

16) $lim_{x\to\infty}\frac{sinx}{x} = 0$ Graph.

17) $lim_{x\to\infty}\frac{\sqrt{x^2+9x}+8x}{7x-19} = lim_{x\to\infty}\frac{x+8x}{7x} = \frac{9}{7}$

18) $lim_{x\to\infty}\left(\sqrt{x^2-9x}+2x\right)\frac{\sqrt{x^2-9x}-2x}{\sqrt{x^2-9x}-2x} = lim_{x\to\infty}\frac{x^2-9x-2x^2}{\sqrt{x^2-9x}-2x} = lim_{x\to\infty}\frac{-x^2}{x-2x} =$

$$lim_{x\to\infty}\frac{-x^2}{x-2x} = lim_{x\to\infty}\frac{-x^2}{-x} = lim_{x\to\infty}x = \infty$$

19) Use the mathematical way to solve this Bill Gates problem.

$$lim_{x\to\infty}\frac{x^2+6x-13}{x^3+9x-12} = lim_{x\to\infty}\frac{\frac{x^2}{x^3}+\frac{6x}{x^3}-\frac{13}{x^3}}{\frac{x^3}{x^3}+\frac{9x}{x^3}-\frac{12}{x^3}} = lim_{x\to\infty}\frac{\frac{1}{x}+\frac{6}{x^2}-\frac{13}{x^3}}{1+\frac{9}{x^2}-\frac{12}{x^3}}$$

$$lim_{x\to\infty}\frac{0+0+0}{1+0+0} = 0$$

20) What are the three options for this limit?

$$lim_{x\to\infty}\frac{fx^h}{px^v}$$

$h > v \quad \infty \qquad h = v \quad \frac{f}{p} \qquad h < v \quad 0$

21) Give an power function for the end behavior for $f(x) = \frac{3x^2+8x-12}{4x+10}$

Do Bill Gates and do not plug in the infinity.

$$f(x) = \frac{3x^2+8x-12}{4x+10} = \frac{3x^2}{4x} = \frac{3x}{4}$$

22) What is the easiest way to find a horizontal asymptote?

Use Bill Gates.

23) Which asymptote (horizontal, vertical, both or neither) can you cross?

Horizontal

24) Draw a function with the following characteristics:

$lim_{x\to\infty}f(x) = \infty, lim_{x\to-\infty}f(x) = 2$

$lim_{x\to-4}f(x) = -3, f(-4) = 7$

$lim_{x\to0^-}f(x) = -\infty, \quad lim_{x\to0^+}f(x) = \infty$

$lim_{x\to5^-}f(x) = 7, \quad lim_{x\to5^+}f(x) = -3$

25) Use the Sandwich Theorem to find $lim_{x\to\infty}\frac{sin^4x}{x^2+1}$.

$$0 \le sin^4x \le 1$$

$$\frac{0}{x^2+1} \le \frac{sin^4x}{x^2+1} \le \frac{1}{x^2+1}$$

$$lim_{x\to\infty}\frac{0}{x^2+1} = 0, lim_{x\to\infty}\frac{1}{x^2+1} = 0$$

$$lim_{x\to\infty}\frac{sin^4x}{x^2+1} = 0$$

26) If $f(x) = \begin{cases} 3x-5 & for -3 \le x < 6 \\ x^2-23 & for\ 6 \le x \end{cases}$, what is $lim_{x\to 6}f(x)$?

$$3(6)-5 = 13$$

$$6^2-23 = 13$$

$$lim_{x\to 6}f(x) = 13$$

27) If $a \ne 0$, then $lim_{x\to a}\frac{x^2-a^4}{x^4-a^8} = lim_{x\to a}\frac{x^2-a^4}{(x^2-a^4)(x^2+a^4)} =$

$$lim_{x\to a}\frac{1}{(x^2+a^4)} = \frac{1}{a^2+a^4}$$

28) For $x \ge 0$, $y = 4$ is the horizontal asymptote for the function $f(x)$. Create the limit that best describes this relationship.

$$lim_{x\to\infty}f(x) = 4$$